Logarithmic Functions

Basics

We can re-write an exponential function as a logarithmic function

$$y = c^x \Leftrightarrow x = \log_c y$$

This is particularly useful when finding the inverse of an exponential function.

Exponential: $y = c^x$

Inverse: $x = c^y$

Re-write as a log: $y = \log_c x$, where $c > 0, c \neq 1$

The log of a number is the exponent to which a base must be raised to get the number.

Ex: $\log_{10} 100 = 2$ means the base (10) must have an exponent of 2 in order to get 100.

Logs can have different bases, but the most common are 10 and e (2.718…)

$\log_{10} x$ can also be written $\log x$ ← If a log has no base written, the base is 10.

$\log_e x$ can also be written $\ln x$ ← If a log is written ln that means the log has a base of e.

Ex: Write each of the following in log form:

a) $10^3 = 1000$

$\log_{10} 1000 = 3$

b) $5^4 = 625$

$\log_5 625 = 4$

c) $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$

$\log_{\frac{1}{2}} \frac{1}{8} = 3$

d) $10^{-4} = 0.0001$

$\log_{10} 0.0001 = -4$

e) $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$

$\log_{\frac{1}{3}} \frac{1}{9} = 2$

f) $a^m = b$

$\log_a b = m$

Switching between exponential and logarithmic forms can help us solve for unknown variables.

Ex: Solve

a) $\log_x 32 = 5$ $x^5 = 32$ $x = \sqrt[5]{32}$ $x = 2$	b) $\log_4 x = 3$ $4^3 = x$ $x = 64$	c) $\log_x \left(\frac{1}{16}\right) = -2$ $x^{-2} = \frac{1}{16}$ $x = \sqrt[-2]{\frac{1}{16}}$ $x = 4$	d) $\log_{\frac{1}{2}} x = 2$ $\left(\frac{1}{2}\right)^2 = x$ $x = \frac{1}{4}$
d) $3^x = 729$ $\log_3 729 = x$ $x = 6$	e) $5^x = 1000$ $\log_5 1000 = x$ $x = 4.2920$	f) $\log_3 -9 = x$ No solution. You cannot take the log of a negative number. This would mean: $3^x = -9$ But there is no possible exponent we can put on 3 to get -9.	

Ex: Simplify

a) $\log_2 8 + \log_4 1024$ $= 3 + 5$ $= 8$	b) $\log 10 + \log 1000 + \log_5 125 - \log_2 1$ $= 1 + 3 + 3 - 0$ $= 7$

Ex: Solve for x

$$\log_4(x^2 + 15x) = 2$$

$$4^2 = x^2 + 15x$$

$$16 = x^2 + 15x$$

$$0 = x^2 + 15x - 16$$

$$0 = (x + 16)(x - 1)$$

$$(x + 16) = 0 \quad or \quad (x - 1) = 0$$

$$x = -16 \quad or \quad x = 1$$

Laws of Logs

There are a number of rules for logs, similar to our laws of exponents. These help us solve or manipulate logs.

Name	Law	Example
Fundamental Law	$\boldsymbol{c^{\log_c m} = m}$ and $\boldsymbol{\log_c c^m = m}$ Why? $c^{\log_c m} = m$ Re-write in log form: $\log_c m = \log_c m$ Why? $\log_c c^m = m$ Re-write in exp. form: $c^m = c^m$	$4^{\log_4 8} = 8$ and $\log_4 4^8 = 8$
Product Law	$\boldsymbol{\log_c mn = \log_c m + \log_c n}$	$\log_5 7 \times 3 = \log_5 7 + \log_5 3$
Quotient Law	$\boldsymbol{\log_c \frac{m}{n} = \log_c m - \log_c n}$	$\log_4 8 \times 5 = \log_4 8 - \log_4 5$
Power Law	$\boldsymbol{\log_c m^n = n \log_c m}$	$\log_8 12^3 = 3 \log_8 12$
Change of Base Law	$\boldsymbol{\log_c m = \frac{\log_n m}{\log_n c}}$ *Where n is any new base you want, so long as it is the same in the numerator and denominator* *(often choose n=10)*	$\log_{20} 4 = \frac{\log_{10} 4}{\log_{10} 20}$

Ex: Expand the following:

a) $\log_3 \left(\frac{5p}{b^2}\right)^8$

$= 8 \log_3 \frac{5p}{b^2}$

$= 8 (\log_3 5p - \log_3 b^2)$

$= 8 (\log_3 5 + \log_3 p - 2 \log_3 b)$

b) $\log_4(a^2 b)$

$= \log_4 a^2 + \log_4 b$

$= 2 \log_4 a + \log_4 b$

c) $\log_5(125a^2b^3)$

$= \log_5 125 + \log_5 a^2 + \log_5 b^3$

$= 3 + 2 \log_5 a + 3 \log_5 b$

Ex: Expand $\log_{10}(x^2-16)+\log_{10}(x+4)^2$

$$=\log_{10}(x-4)(x+4)+2\log_{10}(x+4)$$
$$=\log_{10}(x-4)+\log_{10}(x+4)+2\log_{10}(x+4)$$
$$=\log_{10}(x-4)+3\log_{10}(x+4)$$

Ex: If $\log_2 p=3$ and $\log_2 r=1$, solve $\log_2(8pr)^{16}$

$$\log_2 p=3 \qquad \log_2 r=1$$
$$2^3=p \qquad 2^1=r$$
$$p=8 \qquad r=2$$

So $\log_2(8pr)^{16}$

$$=16\log_2(8\times 8\times 2)$$
$$=16\log_2(128)$$
$$=16\times 7$$
$$=112$$

Ex: Express as a single log and simplify

a) $\log_3 12+\log_3 2$

$$=\log_3(12\times 2)$$
$$=\log_3 24$$

b) $2\log 36-3\log 6$

$$=\log 36^2-\log 6^3$$
$$=\log\frac{36^2}{6^3}$$
$$=\log 6$$

c) $\frac{\log_2 12}{\log_2 3}-\log_3 6$

Change of base law in reverse!

$$=\log_3 12-\log_3 6$$
$$=\log_3\left(\frac{12}{6}\right)$$
$$=\log_3 2$$

d) $\log(x^2-4)-2\log(x+2)$

$$=\log(x+2)(x-2)-\log(x+2)^2$$
$$=\log\frac{(x+2)(x-2)}{(x+2)^2}$$
$$=\log\frac{(x+2)(x-2)}{(x+2)(x+2)}$$
$$=\log\frac{(x-2)}{(x+2)}$$

e) $\log_a t-3\log_a 2t+2\log_a 3t$

$$=\log_a t-\log_a(2t)^3+\log_a(3t)^2$$
$$=\log_a t-\log_a 8t^3+\log_a 9t^2$$
$$=\log_a\frac{t}{8t^3}+\log_a 9t^2$$
$$=\log_a\frac{9t^3}{8t^3}$$
$$=\log_a\frac{9}{8}$$

Simplifying Logs with Different Bases

We can use the change of base law to simplify logs with different bases.

Remember the change of base law:

$$\log_c m = \frac{\log_n m}{\log_n c}$$

where n is any new base you want, so long as it is the same in the numerator and denominator

Ex: Simplify:

a) $\log_3 x + \log_{27} x$

$$= \log_3 x + \frac{\log_3 x}{\log_3 27}$$

Use change of base law (on either log) to make bases the same

$$= \log_3 x + \frac{\log_3 x}{3}$$

Remember $\log_3 27$ is a number (3)

$$= (\log_3 x)\left(1 + \frac{1}{3}\right)$$

Factor out a common term

$$= (\log_3 x)\left(\frac{4}{3}\right)$$

Simplify the term in brackets

$$= \frac{4}{3}(\log_3 x)$$

Re-write

b) $\log_5 x + \log_8 x$

$$= \log_5 x + \frac{\log_5 x}{\log_5 8}$$

$$= (\log_5 x)(1 + 0.774)$$

$$= 1.774 \log_5 x$$

If your calculator cannot find log of bases other than 10 and e, you can always use the change of base law on the denominator:

$$\log_5 8 = \frac{\log_{10} 8}{\log_{10} 5} = 1.292$$

Ex: Simplify

a) $\log_3 x - \ln x$

$$= \log_3 x - \frac{\log_3 x}{\log_3 e}$$

$$= (\log_3 x)(1 - \frac{1}{\log_3 e})$$

$$= -0.099 \log_3 x$$

b) $\log_3 x + \log_{27} y + \log_9 z$

$$= \log_3 x + \frac{\log_3 y}{\log_3 27} + \frac{\log_3 z}{\log_3 9}$$

$$= \log_3 x + \frac{\log_3 y}{3} + \frac{\log_3 z}{2}$$

$$= \log_3 x + \frac{1}{3}\log_3 y + \frac{1}{2}\log_3 z$$

$$= \log_3 x + \log_3 y^{1/3} + \log_3 z^{1/2}$$

$$= \log_3 x\, y^{1/2} z^{1/2}$$

Log Function Basic and Transformed

Basic Function

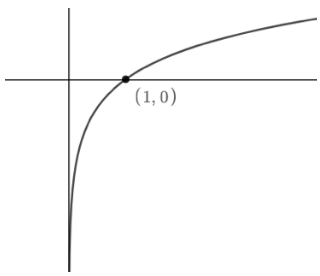
A log function is the inverse of an exponential function, and the rule for the basic function is:

$$f(x) = \log_c x$$

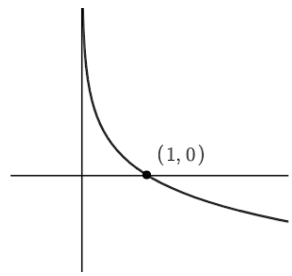
where $c \neq 1, c > 0$

- Because you can't take the log of a negative number, this function has a vertical asymptote (at $x = 0$ for the basic function)
- Since $\log_c 1 = 0$, the basic function passes through the point (1, 0)
- Domain of the basic function: $]0, \infty[$
- Range: $]-\infty, \infty[$

If $c > 1$



If $0 < c < 1$



Transformed Function

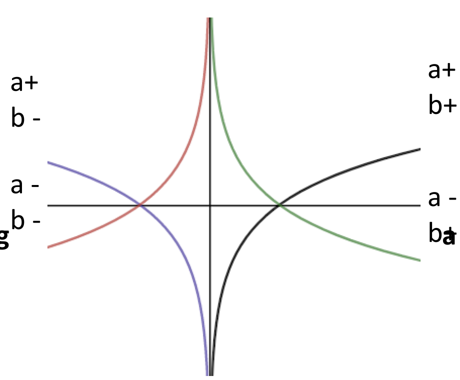
Just like all other functions we've looked at this year, we can add the parameters a, b, h, and k

$$f(x) = a\log_c(b(x-h)) + k$$

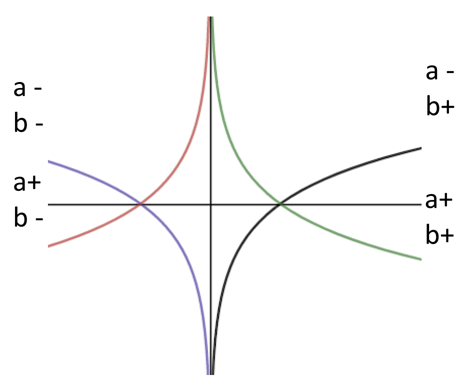
where $c \neq 1, c > 0, a \neq 0, b \neq 0$

- Vertical asymptote at $x = h$
- An x-intercept (zero) will always exist.
- A y-intercept may or may not exist
  - If b and h have the same sign, no y-intercept
  - If b and h have opposite signs, a y-intercept exists
- Domain:
  - If $b > 0$, $]h, \infty[$
  - If $b < 0$, $]-\infty, h[$
- Range: $]-\infty, \infty[$

If $c > 1$



If $0 < c < 1$



Sketching a Logarithmic Function

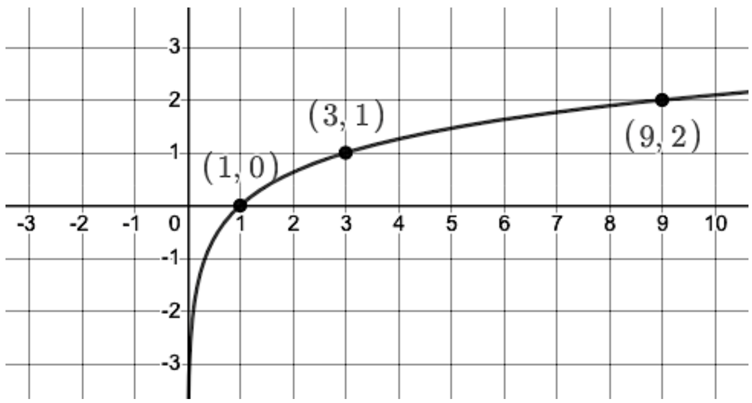
To sketch a logarithmic function:

- Determine the asymptote
- Determine the direction of the curve and select x accordingly
- Create a table of values
- Connect the dots

Ex: Sketch the function $f(x) = \log_3 x$

Asymptote: $x = 0$

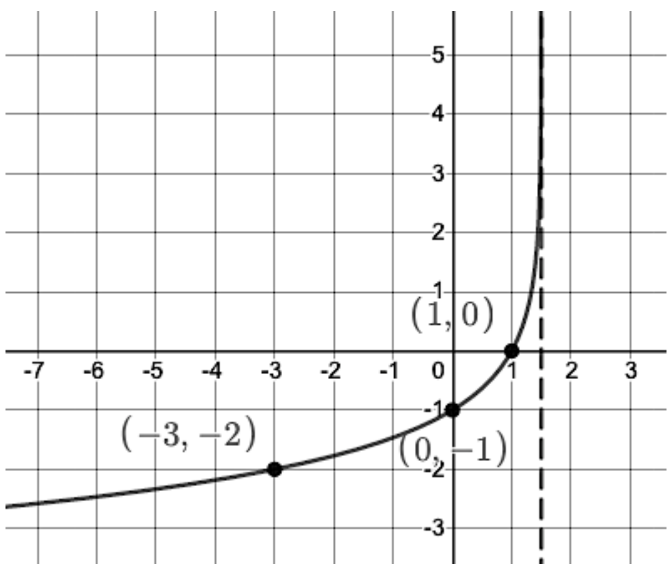
x	y
1	0
3	1
9	2



Ex: Sketch the function $f(x) = -\log_3\left(-2(x - 1.5)\right)$

Asymptote: $x = 1.5$

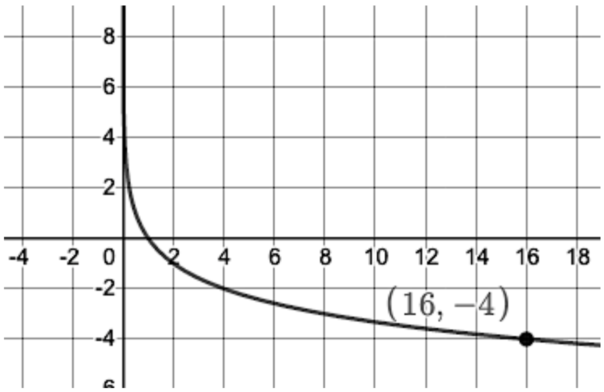
x	y
1	0
0	-1
-3	-2



Finding the Rule of Logarithmic Functions

To find the rule of logarithmic functions, plug in what you know and solve for the remaining variable(s).

Ex: A logarithmic function is sketched below. Find the rule of the function in the form $f(x) = \log_c x$



$$f(x) = \log_c x$$

$$-4 = \log_c 16$$

$$c^{-4} = 16$$

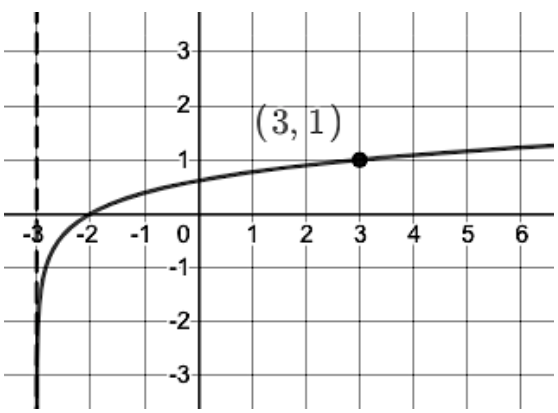
$$c = \sqrt[-4]{16}$$

$$c = \frac{1}{2}$$

Therefore $f(x) = \log_{\frac{1}{2}} x$

Ex: A logarithmic function is sketched below. Find the rule of the function in the form

$$f(x) = \log_c(x - h)$$



Asymptote at $x = -3$, so $h = -3$

$$f(x) = \log_c(x - h)$$

$$f(x) = \log_c(x + 3)$$

$$1 = \log_c(3 + 3)$$

$$1 = \log_c(6)$$

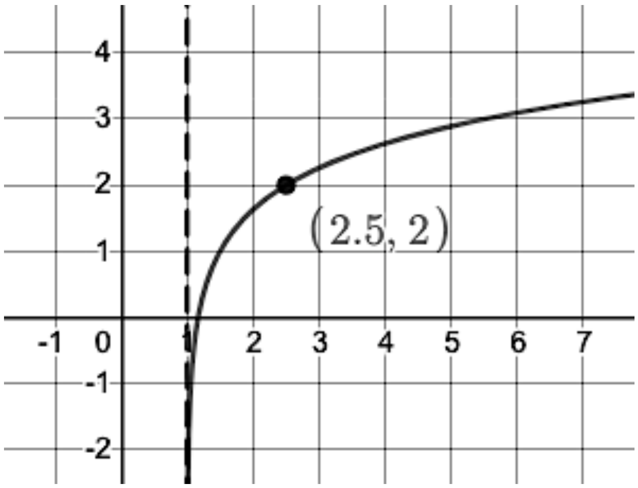
$$c^1 = 6$$

$$c = 6$$

Therefore $f(x) = \log_6(x + 3)$

Ex: A logarithmic function is sketched below. Find the rule of the function in the form

$$f(x) = \log_3 b(x - h)$$



Asymptote at $x = 1$, so $h = 1$

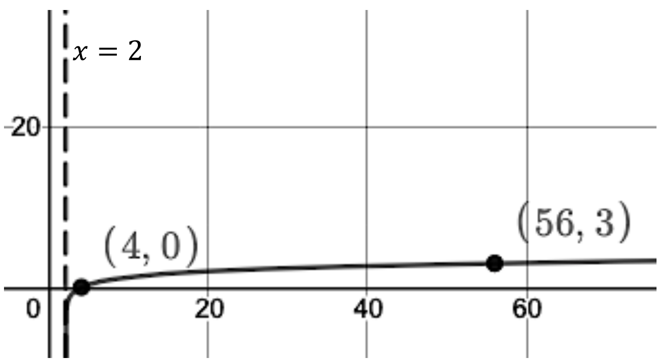
$$f(x) = \log_3 b(x - h)$$
$$f(x) = \log_3 b(x - 1)$$
$$2 = \log_3 b(2.5 - 1)$$
$$2 = \log_3(1.5b)$$
$$3^2 = 1.5b$$
$$9 = 1.5b$$
$$6 = b$$

Therefore $f(x) = \log_3 6(x - 1)$

Additionally, using the laws of logs, both a and k can be integrated into the b term. So when finding the rule of a logarithmic function, we can always use:

$f(x) = \log_c\big(b(x - h)\big)$ **unless specified otherwise in the question.*

Ex: Find the rule of the logarithmic function sketched below



Asymptote at $x = 2$, so $h = 2$

$$f(x) = \log_c b(x - h)$$

Using point (4, 0)

$$0 = \log_c b(4 - 2)$$
$$0 = \log_c b(2)$$
$$c^0 = 2b$$
$$1 = 2b$$
$$b = \frac{1}{2}$$

Using point (56, 3)

$$3 = \log_c \frac{1}{2}(56 - 2)$$
$$3 = \log_c \frac{1}{2}(54)$$
$$3 = \log_c 27$$
$$c^3 = 27$$
$$c = 3$$
$$\therefore f(x) = \log_3 \frac{1}{2}(x - 2)$$

Solving Logarithmic Functions

To solve for y given x, plug in x and solve

**you may have to use a change of base law depending on your calculator*

Ex:

a) Solve $f(x) = 2.5\log_2 4(x-2) - 10$

when $x = 6$

$$f(x) = 2.5\log_2 4(x-2) - 10$$
$$f(6) = 2.5\log_2 4(6-2) - 10$$
$$f(6) = 2.5\log_2 4(4) - 10$$
$$f(6) = 2.5\log_2 16 - 10$$
$$f(6) = 2.5 \times 4 - 10$$
$$f(6) = 10 - 10$$
$$f(6) = 0$$

b) Solve $f(x) = 2\log_3 2(x-5)$

when $x = 8$

$$f(x) = 2\log_3 2(x-5)$$
$$f(8) = 2\log_3 2(8-5)$$
$$f(8) = 2\log_3 2(3)$$
$$f(8) = 2\log_3 6$$
$$f(8) = 2 \times 1.6309$$
$$f(8) = 3.2618$$

To solve for x given y:

1) State restrictions (you cannot take the log of a negative number or 0)
2) Apply laws of logs to write equation as a single log
3) Isolate the log
4) Change to exponential form
5) Solve for x and check against restrictions

Ex:

a) Solve $2\log_3 2(x+5) = 6$

Restrictions:

$2(x+5) > 0$

$(x+5) > 0$

$x > -5$

$$2\log_3 2(x+5) = 6$$
$$\log_3 2(x+5) = 3$$
$$3^3 = 2(x+5)$$
$$27 = 2(x+5)$$
$$13.5 = x+5$$
$$8.5 = x$$

fits with restrictions

b) Solve $\ln(5-x) + \ln(x-3) = 0$

Restrictions:

$(5-x) > 0 \ and \ (x-3) > 0$

$x < 5 \ \ and \ x > 3$

$$\ln(5-x) + \ln(x-3) = 0$$
$$\ln(5-x)(x-3) = 0$$
$$e^0 = (5-x)(x-3)$$
$$1 = 5x - 15 - x^2 + 3x$$
$$0 = -x^2 + 8x - 16$$
$$0 = -(x-4)(x-4)$$
$$x = 4$$

Fits with restrictions

Ex:

a) Solve $\log x^2 = \log x - 2$

Restrictions:

$x^2 > 0 \quad and\ x > 0$

$x > 0$

$$\log x^2 = \log x - 2$$

$$\log x^2 - \log x = -2$$

$$\log \frac{x^2}{x} = -2$$

$$\log x = -2$$

$$10^{-2} = x$$

$$x = 0.01$$

Fits with restrictions

b) Solve $\log(x+2) = 1 - \log(x-1)$

Restrictions:

$x + 2 > 0 \quad and\ x - 1 > 0$

$x > -2\ and\ x > 1$

$$\log(x+2) = 1 - \log(x-1)$$

$$\log(x+2) + \log(x-1) = 1$$

$$\log(x+2)(x-1) = 1$$

$$10^1 = (x+2)(x-1)$$

$$10 = x^2 + x - 2$$

$$0 = x^2 + x - 12$$

$$0 = (x+4)(x-3)$$

$$x = -4\ and\ x = 3$$

No Yes

$$x = 3$$

Ex:

a) Solve $\log_8(2x-7) = \log_8(5-x)$

Restrictions:

$2x - 7 > 0\ and\ 5 - x > 0$

$x > 3.5\ and\ x < 5$

$$\log_8(2x-7) = \log_8(5-x)$$

$$\log_8(2x-7) - \log_8(5-x) = 0$$

$$\log_8 \frac{2x-7}{5-x} = 0$$

$$8^0 = \frac{2x-7}{5-x}$$

$$1 = \frac{2x-7}{5-x}$$

$$5 - x = 2x - 7$$

$$12 = 3x$$

$$x = 4$$

Fits with restrictions

b) Solve $\log_3(x+4) - \log_3 x = \log_2 4$

Restrictions

$x + 4 > 0\ and\ x > 0$

$x > -4\ and\ x > 0$

$$\log_3(x+4) - \log_3 x = \log_2 4$$

$$\log_3 \frac{x+4}{x} = 2$$

$$3^2 = \frac{x+4}{x}$$

$$9 = \frac{x+4}{x}$$

$$9x = x + 4$$

$$8x = 4$$

$$x = 0.5$$

Fits with restrictions

Solving Logarithmic Inequalities

To solve logarithmic inequalities:

1) change the inequalities to $=$
2) use a test point or graph to determine the appropriate interval

Ex: Solve $-18\log_9(-4x) \geq -9$

Solve

$$-18\log_9(-4x) \geq -9$$
$$-18\log_9(-4x) = -9$$
$$\log_9(-4x) = 0.5$$
$$9^{0.5} = -4x$$
$$3 = -4x$$
$$x = -4/3$$

Test Point

$c > 1, a+, b-, h = 0$ so domain: $]-\infty, 0[$

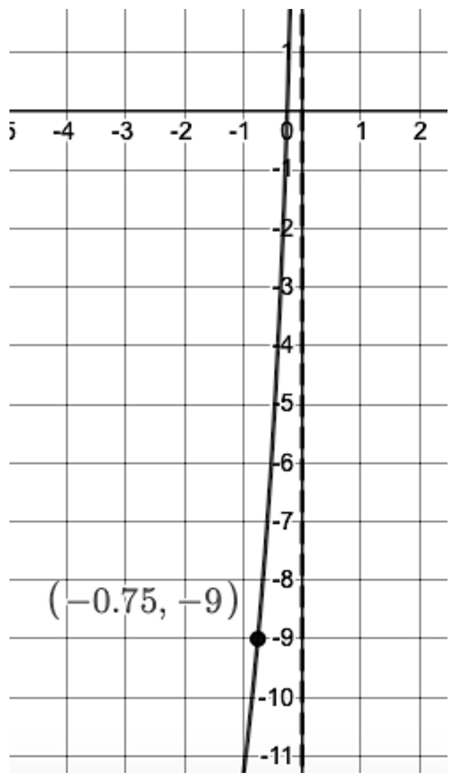
Test $x = -1$

$$-18\log_9(-4x) \geq -9$$
$$-18\log_9(-4 \times -1) \geq -9$$
$$-18\log_9(4) \geq -9$$
$$-11.3567 \geq 0$$

FALSE

Sketch

$c > 1, a+, b-, h = 0$



$$\therefore -18\log_9(-4x) \geq -9 \ over \left[-\frac{3}{4}, 0\right[$$

Ex: Given $f(x) = -\log_{0.5}(x) - 2$ and $g(x) = \log_4(2x-4) + 3$

Determine the interval over which $f(x) > g(x)$

$$-\log_{0.5}(x) - 2 = \log_4(2x-4) + 3$$

$$-\log_{0.5}(x) - \log_4(2x-4) = 5$$

$$\log_{0.5}(x)^{-1} - \log_4(2x-4) = 5$$

$$\frac{\log_4 x^{-1}}{\log_4 0.5} - \log_4(2x-4) = 5$$

$$\frac{\log_4 x^{-1}}{-\frac{1}{2}} - \log_4(2x-4) = 5$$

$$-2\log_4 x^{-1} - \log_4(2x-4) = 5$$

$$\log_4 x^2 - \log_4(2x-4) = 5$$

$$\log_4 \frac{x^2}{2x-4} = 5$$

$$4^5 = \frac{x^2}{2x-4}$$

$$1024 = \frac{x^2}{2x-4}$$

$$2048x - 4096 = x^2$$

$$0 = x^2 - 2048x + 4096$$

USE QUADRATIVE FORMULA TO SOLVE

$$x = \frac{2048 \pm \sqrt{2048^2 - 4(1)(4096)}}{2(1)}$$

$$x = 2045.996 \ and \ x = 2.002$$

SKETCH TO SOLVE

$$f(x) > g(x) \ over \ ]2, 2.002[\cup]2045.996, \infty[$$

Solving Exponential Functions Using Logs

When you have an exponential function and you are trying to solve for the exponent:

1) Isolate the base and exponent
2) Write in log form if possible, or take the log of both sides
3) Apply laws of logs and solve

Ex: Solve the following

a) $-30(0.9)^{0.5x} + 15 = 0$

$$-30(0.9)^{0.5x} = -15$$
$$(0.9)^{0.5x} = 0.5$$
$$\log_{0.9} 0.5 = 0.5x$$
$$6.5788 = 0.5x$$
$$x = 13.1576$$

b) $4^{2x-3} = 5^x$

$$\log 4^{2x-3} = \log 5^x$$
$$(2x-3)\log 4 = x \log 5$$
$$2x - 3 = x\left(\frac{\log 5}{\log 4}\right)$$
$$2x - 3 = 1.1610x$$
$$0.839x = 3$$
$$x = 3.5757$$

c) $2 \times 4^{2x-3} + 6 = 5^x + 6$

$$2 \times 4^{2x-3} = 5^x$$
$$\log(2 \times 4^{2x-3}) = \log 5^x$$
$$\log 2 + \log 4^{2x-3} = x \log 5$$
$$0.301 + (2x-3)(\log 4) = 0.699x$$
$$0.301 + (2x-3)(0.602) = 0.699x$$
$$0.301 + 1.204x - 1.806 = 0.699x$$
$$0.505x = 1.505$$
$$x = 2.9801$$

Ex: There are 2 types of bacteria in separate petri dishes. When are the populations the same?

Type 1: Initially there are 5 bacteria. The population doubles every week

Type 2: Initially there are 20 bacteria. The population increases by 1% every day.

Type 1: $f(x) = 5(2^x)$

Type 2: $g(x) = 20(1.01^{7x})$

Where x is measured in weeks

$$f(x) = g(x)$$
$$5(2^x) = 20(1.01^{7x})$$
$$(2^x) = 4(1.01^{7x})$$
$$\log 2^x = \log 4 + \log 1.01^{7x}$$
$$x(0.3010) = 0.6021 + 7x(0.0043)$$
$$0.3010x - 0.0301x = 0.6021$$
$$0.2709x = 0.6021$$
$$x = 2.22\ weeks$$

Finding the Inverse of Exponential and Logarithmic Functions

To find the inverse of a logarithmic function:

1) Swap the x and y
2) Isolate the log
3) Write in exponential form
4) Solve for y

Ex: Find the rule of the inverse function

a) $f(x) = \log_{\frac{1}{4}} x$

$$x = \log_{1/4} y$$

$$\left(\frac{1}{4}\right)^x = y$$

$$f^{-1}(x) = \left(\frac{1}{4}\right)^x$$

b) $f(x) = 4\log[2(x+5)] - 2$

$$x = 4\log[2(y+5)] - 2$$

$$\frac{x+2}{4} = \log[2(y+5)]$$

$$10^{\left(\frac{x+2}{4}\right)} = 2(y+5)$$

$$\frac{1}{2}\left(10^{\frac{1}{4}(x+2)}\right) = y+5$$

$$y = \frac{1}{2}\left(10^{\frac{1}{4}(x+2)}\right) - 5$$

$$f^{-1}(x) = \frac{1}{2}\left(10^{\frac{1}{4}(x+2)}\right) - 5$$

c) $f(x) = -2\ln -(x-2) + 4$

$$x = -2\ln -(y-2) + 4$$

$$\frac{x-4}{-2} = \ln -(y-2)$$

$$e^{\frac{x-4}{-2}} = -(y-2)$$

$$-(e^{\frac{x-4}{-2}}) = y - 2$$

$$y = -\left(e^{\frac{x-4}{-2}}\right) + 2$$

$$f^{-1}(x) = -\left(e^{\frac{x-4}{-2}}\right) + 2$$

To find the inverse of an exponential function:

1) Swap x and y
2) Isolate the base and the exponent
3) Write in exponential form
4) Solve for y

Ex: Find the inverse of the following functions:

a) $f(x) = -2(10)^{x+2}$

$$x = -2(10)^{y+2}$$

$$\frac{x}{-2} = (10)^{y+2}$$

$$y + 2 = \log_{10}\left(\frac{x}{-2}\right)$$

$$y = \log_{10}\left(\frac{x}{-2}\right) - 2$$

$$f^{-1}(x) = \log_{10}\left(\frac{x}{-2}\right) - 2$$

b) $f(x) = \frac{1}{2}(5)^{2(x-1)} + 6$

$$x = \frac{1}{2}(5)^{2(y-1)} + 6$$

$$x - 6 = \frac{1}{2}(5)^{2(y-1)}$$

$$2(x-6) = (5)^{2(y-1)}$$

$$2(y-1) = \log_5 2(x-6)$$

$$y - 1 = \frac{1}{2}\log_5 2(x-6)$$

$$y = \frac{1}{2}\log_5 2(x-6) + 1$$

$$f^{-1}(x) = \frac{1}{2}\log_5 2(x-6) + 1$$

One More Complication (annual interest rate compounded monthly)

Using our original equation, we can change time to fit interest rate (so both are annual, for example), but we don't know how to resolve the fact that the compounding period is different than the interest rate period.

So we need another formula:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Where:

A is final amount

P is principal (starting amount)

r is annual interest rate as a decimal

n is the number of compoundings per year

t is time in years

Ex: You want \$10 000 to buy a used car in 3 years. You will invest money in an account earning an annual interest rate of 3.5% compounded monthly. How much do you need to invest to have the money you want?

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$10\ 000 = P\left(1 + \frac{0.035}{12}\right)^{12\times 3}$$

$$10\ 000 = P(1.0029)^{36}$$

$$10\ 000 = P(1.10988)$$

$$9009.98 = P$$

Therefore, you should invest \$9009.98.