Solving Trigonometric Equations

When we are asked to solve a trigonometric equation, we are going to have to combine what we've learned so far (trig ratios, trig identities, the unit circle, etc.) with algebra (factoring, etc.) to solve for x or θ.

Ex: Solve for x

$$cos^2x-(1-cos^2x)=2-5\cos x$$
$$cos^2x-(1-cos^2x)-2+5\cos x=0$$
$$cos^2x-1+cos^2x-2+5\cos x=0$$
$$2cos^2x-3+5\cos x=0$$
$$2cos^2x+5\cos x-3=0$$
$$(2\cos x-1)(\cos x+3)=0$$
$$(2\cos x-1)=0 \quad or \quad (\cos x+3)=0$$
$$\cos x=\frac{1}{2} \quad or \quad \cos x=-3$$

$$x=\frac{\pi}{3}\ and\ x=\frac{5\pi}{3} \qquad\qquad no\ solutions$$
$$\therefore\ x=\frac{\pi}{3}\ and\ x=\frac{5\pi}{3}$$

Ex: Solve for x

$$4\ sin^3x-\sin x=0$$
$$(\sin x)(4sin^2x-1)=0$$
$$(\sin x)(2\sin x-1)(2\sin x+1)=0$$
$$(\sin x)=0 \quad or \quad (2\sin x-1)=0 \quad or \quad (2\sin x+1)=0$$

$$x=0,\pi,2\pi \qquad \sin x=\frac{1}{2} \qquad \sin x=-\frac{1}{2}$$
$$x=\frac{\pi}{6},\frac{5\pi}{6} \qquad \sin x=\frac{7\pi}{6},\frac{11\pi}{6}$$

$$\therefore\ x=0,\frac{\pi}{6},\frac{5\pi}{6},\pi,\frac{7\pi}{6},\frac{11\pi}{6},2\pi$$

Ex: Solve for θ

$$(\cos\theta - 2)^2 = 2$$
$$\cos\theta - 2 = \pm\sqrt{2}$$
$$\cos\theta = \pm\sqrt{2} + 2$$

$$\cos\theta = \sqrt{2} + 2 \quad or \quad \cos\theta = -\sqrt{2} + 2$$
$$\theta_R = 0.9449 \qquad \theta_R = ERROR$$

$$\theta_1 = 0.9449 \quad and\ \theta_2 = 5.3382$$

$$\therefore \theta_1 = 0.9449 \quad and\ \theta_2 = 5.3382$$