

Rational Functions Practice Questions

Rational Functions: Basics

1) For each of the rational functions below, determine the equations of both asymptotes and state the domain and range.

a. $f(x) = \frac{3x+8}{x-2}$

Asymptotes: $x = 2$ and $y = 3$

D: $]-\infty, 2[\cup]2, +\infty[$

R: $]-\infty, 3[\cup]3, +\infty[$

b. $g(x) = \frac{3}{7x+14} - 5$

Asymptotes: $x = -2$ and $y = -5$

D: $]-\infty, -2[\cup]-2, +\infty[$

R: $]-\infty, -5[\cup]-5, +\infty[$

c. $h(x) = \frac{2x+6}{3-x}$

Asymptotes: $x = 3$ and $y = -2$

D: $]-\infty, 3[\cup]3, +\infty[$

R: $]-\infty, -2[\cup]-2, +\infty[$

a. $i(x) = \frac{1}{x-3} + 5$

Asymptotes: $x = 3$ and $y = 5$

D: $]-\infty, 3[\cup]3, +\infty[$

R: $]-\infty, 5[\cup]5, +\infty[$

2) Write each of the following rules in the form $f(x) = \frac{a}{x-h} + k$

a) $f(x) = \frac{5x+8}{2x+1}$

$$y = \frac{5.5}{2x+1} + 2.5$$

OR

$$y = \frac{11}{4(x+0.5)} + 2.5$$

b) $g(x) = \frac{4-2x}{1-x}$

$$y = \frac{2}{-x+1} + 2$$

c) $i(x) = \frac{4x-5}{3-x}$

$$y = \frac{7}{-x+3} - 4$$

d) $j(x) = \frac{-4x+3}{2x+1}$

$$y = \frac{5}{2x+1} - 2$$

3) Given $f(x) = \frac{-4}{x+2}$ and $g(x) = \frac{3x}{2x+4}$, find the rule that corresponds to:

a) $f + g$

$$h(x) = \frac{3x - 8}{2x + 4}$$

b) $f \times g$

$$h(x) = \frac{-12}{2x^2 + 8x + 8}$$

d) $\frac{g}{f}$

$$h(x) = \frac{3x^2 + 6x}{-8x - 16}$$

e) $g(f(x))$

$$h(x) = -\frac{3}{x}$$

e) $g - f$

$$h(x) = \frac{3x + 8}{2x + 4}$$

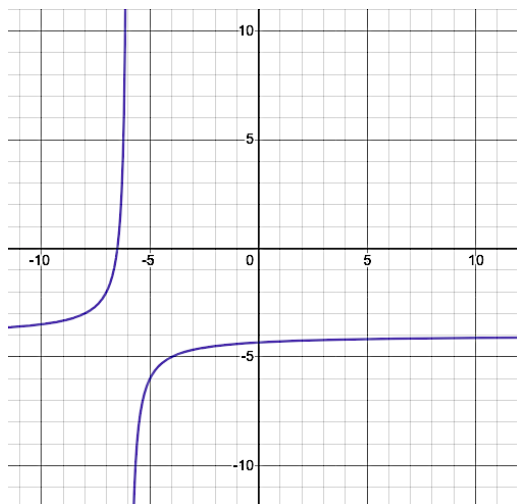
f) $f \circ g$

$$h(x) = \frac{-8x - 16}{7x + 8}$$

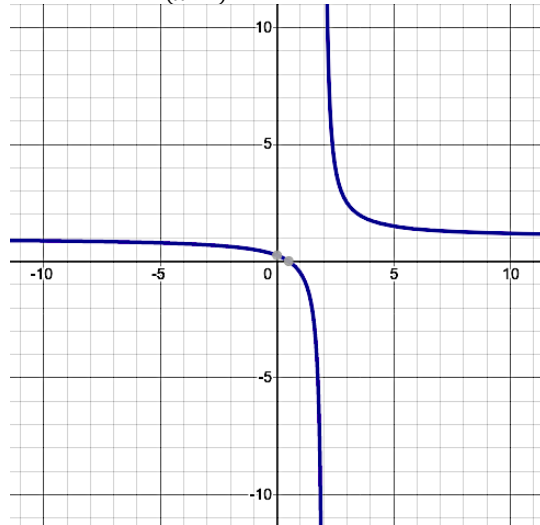
Rational Functions: Graphing

4) Graph each of the following functions (including asymptotes):

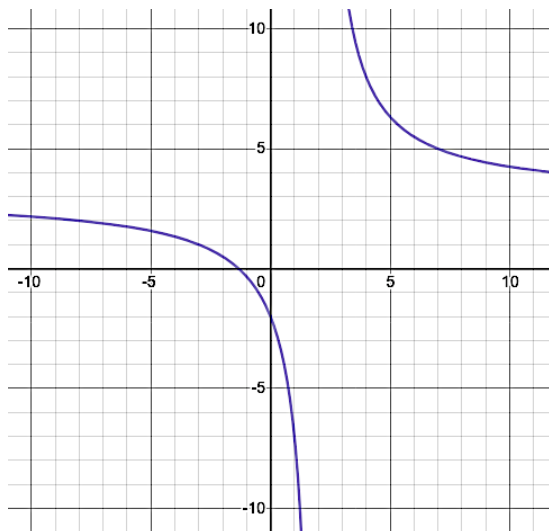
a) $f(x) = \frac{-2}{x+6} - 4$



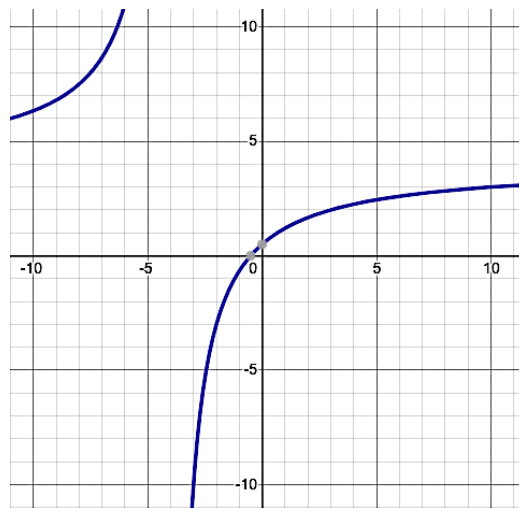
b) $g(x) = \frac{3}{2(x-2)} + 1$



c) $h(x) = \frac{3x+4}{x-2}$



d) $i(x) = \frac{8x+4}{8+2x}$



Rational Functions: Finding the Rule

5) Find the rule of each of the following rational functions described below.

a) Asymptotes at $x = 2$ and $y = 4$ and passing through the point $(0, 2.5)$

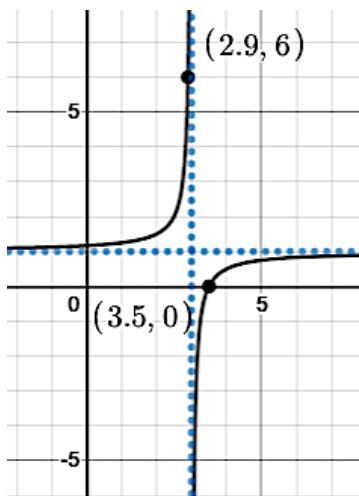
$$f(x) = \frac{3}{x-2} + 4$$

b) Asymptotes at $x = -2$ and $y = -4$ and passing through the point $(-3.25, 0)$

$$f(x) = -\frac{5}{x+2} - 4$$

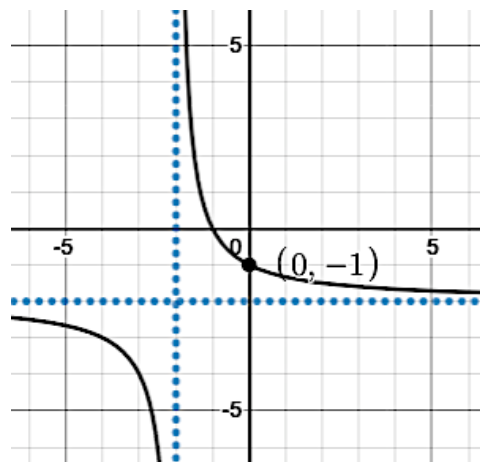
6) Determine the rule of the rational functions graphed below.

a)



$$f(x) = -\frac{0.5}{x-3} + 1$$

b)



$$f(x) = \frac{2}{x+2} - 2$$

Rational Functions: Finding the Rule from Words

7) During a chemical experiment, 6 g of solute are mixed with 800 mL of solvent. Subsequently, the quantities of solute and solvent increase at a continuous rate of 1.7g/min for the solute and 50 mL for the solvent. Given $concentration = \frac{solute}{solvent}$, write an equation to determine the concentration of the solution over time.

$$C = \frac{1.7x + 6}{50x + 800}$$

8) To go on a field trip to an insectarium, students must pay their share of the \$300 transportation cost, plus an \$8 admission charge per student. Write a rule to determine the amount each student must pay based on the number of students taking part in the activity.

$$C = \frac{8x + 300}{x}$$

9) A company pays its president an annual salary of \$250 000, and each of its other x employees \$35 000. Write a rule that represents the company's average salary (including the president's).

$$S = \frac{35000x + 250000}{x + 1}$$

Rational Functions: Solving Equalities

10) Solve the following equations

$$a) -9 = \frac{4}{3-x} + 2$$

$$x = 3.367$$

$$b) 7 = \frac{7x-25}{x-4}$$

No solution

$$c) 2 = \frac{3}{2x+6} - 5$$

$$x = -2.786$$

$$d) \frac{3x+8}{x-5} = 10$$

$$x = 8.2857$$

$$e) 0.25 = \frac{2x-5}{x+1}$$

$$x = 3$$

$$f) 56 = \frac{3x}{2x+2}$$

$$x = -1.0275$$

11) Find the zero and initial value of the following function:

$$f(x) = \frac{4}{3x+8} - 2$$

Initial Value: $x = -2$

Zero: $y = -1.5$

12) Given $f(x) = \frac{3x+2}{x-1}$ and $g(x) = 2x + 3$, determine when $f(x) = g(x)$

$$x = -1.1583 \text{ and } x = 2.1583$$

Rational Functions: Solving Inequalities

13) Solve the following inequalities

a) $7 > \frac{5}{8-x} + 1$

$]-\infty, 7.16[\cup]8, +\infty[$

b) $\frac{1}{4x+1} < 10$

$]-\infty, -0.25[\cup]-0.225, +\infty[$

c) $5 \leq \frac{5}{9x+2} - 4$

$]-0.2, -0.1605]$

d) $4 \geq \frac{2x+6}{8-3x}$

$]-\infty, 1.857] \cup]2.67, +\infty[$

e) $0.75 \geq \frac{2x+8}{7x-1}$

$]-\infty, 0.1429[\cup]2.6923, +\infty[$

f) $\frac{7}{8} \leq \frac{2}{3x+9}$

$]-3, -2.2381]$

Rational Functions: Word Problems

14) During the production of an alloy, the temperature T (in $^{\circ}\text{C}$) of a melting metal varies according to the rule $T = \frac{1000x}{40+x}$ where x corresponds to the time (in min.). What is the maximum temperature reached by this melting metal? Explain your answer.

Max temperature is just under 1000°C

15) Atmospheric pressure drops as altitude increases. The function $f(x) = \frac{530}{x+5}$ is used to obtain the approximate atmospheric pressure in kilopascals based on the altitude in kilometers. What is the atmospheric pressure at sea level according to this approximation?

At sea level, the atmospheric pressure is 106 kilopascals

16) Michelle is playing basketball. Out of her last 40 free throw attempts, she has made a total of 24 of them. The function $f(x) = \frac{24+x}{40+x}$ represents her average if she makes x successful free throw baskets in a row starting now. How many consecutive free throw baskets must she make to have an average of 75%?

She must make 24 baskets in a row to have an average of 75%

Rational Functions: Inverse

17) Determine the inverse of the following functions, and state the domain and range of the inverse.

$$\text{a) } f(x) = \frac{2}{x+4} - 6$$

$$f^{-1}(x) = \frac{2}{x+6} - 4$$

$$D:]-\infty, -6[\cup]-6, +\infty[$$

$$R:]-\infty, -4[\cup]-4, +\infty[$$

$$\text{b) } g(x) = \frac{3x+8}{7x+10}$$

$$f^{-1}(x) = \frac{-10x+8}{7x-3}$$

$$D:]-\infty, \frac{3}{7}[\cup]\frac{3}{7}, +\infty[$$

$$R:]-\infty, \frac{10}{7}[\cup]\frac{10}{7}, +\infty[$$

$$\text{c) } h(x) = \frac{4}{5(x-1)} - 2$$

$$f^{-1}(x) = \frac{4}{5(x+2)} + 1$$

$$D:]-\infty, -2[\cup]-2, +\infty[$$

$$R:]-\infty, 1[\cup]1, +\infty[$$