

## Sample Problems

Solve each of the following equations.

1.)  $\sin x = -\sin^2 x$

2.)  $2\cos^2 x - 5\cos x = 3$

3.)  $3(1 - \sin x) = 2\cos^2 x$

4.)  $\sin x = -\cos x$

5.)  $\tan^2 x = \frac{1}{3}$

6.)  $\cos x \sin x = \cos x$

7.)  $\tan x = \tan^2 x$

8.)  $2\cos x - \sin x + 2\cos x \sin x = 1$

9.)  $\cos x = 1 + \sin^2 x$

10.)  $7\sin x + 5 = 2\cos^2 x$

## Practice Problems

Solve each of the following equations.

1.)  $1 + \sin x = 2\cos^2 x$

2.)  $-3\cos x + 3 = 2\sin^2 x$

3.)  $\cos^3 x = \cos^2 x$

4.)  $2\cos^2 x - \cos x = 3$

5.)  $2\sin^2 x = \cos x + 1$

6.)  $2\cos^2 x + 3\sin x = 3$

7.)  $\sec^2 x = 4$

8.)  $\tan x \sin^2 x = \frac{3}{4} \tan x$

9.)  $\frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x} = 4$

10.)  $\sin^2 x - \frac{1}{2} \cos x + \cos x \sin^2 x = \frac{1}{2}$

11.)  $\cot x = \cos x$

12.)  $\tan x - \sqrt{2} = \frac{1}{\tan x + \sqrt{2}}$

ANSWER KEY: TRIGONOMETRIC EQUATIONS  
Domains [0, 2π]

SAMPLE

①  $\sin x = -\sin^2 x$

$$\sin^2 x + \sin x = 0$$

$$\sin x (\sin x + 1) = 0$$

$$\sin x = 0 \quad x = 0, \pi, 2\pi$$

$$\sin x = -1 \quad x = \frac{3\pi}{2}$$

$$x = 0, \pi, \frac{3\pi}{2}, 2\pi$$

②  $2\cos^2 x - 5\cos x = 3$

$$2\cos^2 x - 5\cos x - 3 = 0$$

$$(2\cos x + 1)(\cos x - 3) = 0$$

$$\cos x = -\frac{1}{2}$$

$$\cos x = 3$$

no solutions

$$x = \frac{2\pi}{3}$$

$$x = \frac{4\pi}{3}$$

$$x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

③  $3(1 - \sin x) = 2\cos^2 x$

$$3 - 3\sin x = 2(1 - \sin^2 x)$$

$$3 - 3\sin x = 2 - 2\sin^2 x$$

$$2\sin^2 x - 3\sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x - 1) = 0$$

$$\sin x = \frac{1}{2} \quad x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\sin x = 1 \quad x = \frac{\pi}{2}$$

$$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$$

④  $\sin x = -\cos x$

Two spots on the unit circle  
where  $\sin x = -\cos x$ , but have opposite signs.

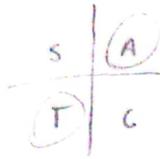
$$\frac{3\pi}{4} \text{ and } \frac{7\pi}{4}$$

SAMPLE PROBLEMS <cont.>

5)  $\tan^2 x = \frac{1}{3}$

$\sqrt{\tan^2 x} = \sqrt{\frac{1}{3}}$

$\tan x = \frac{1}{\sqrt{3}}$



$x = \frac{\pi}{3} \text{ and } \frac{4\pi}{3}$

We know that  $\frac{\sqrt{3}}{2} \div \frac{1}{2} = \frac{1}{\sqrt{3}}$  ( $\frac{\sin}{\cos}$ )

$\therefore \sin \theta = \frac{\sqrt{3}}{2}$  and  $\cos \theta = \frac{1}{2}$

Two places on the circle:  $\frac{\pi}{3}, \frac{4\pi}{3}$

6)  $\cos x \sin x = \cos x$

$\cos x \sin x - \cos x = 0$

$\cos x (\sin x - 1) = 0$

$\cos x = 0 \rightarrow \text{at } \frac{\pi}{2}, \frac{3\pi}{2}$

$\sin x = 1 \rightarrow \frac{\pi}{2}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

7)  $\tan x = \tan^2 x$

$0 = \tan^2 x - \tan x$

$0 = \tan x (\tan x - 1)$

$0 = \tan x \rightarrow x = 0, \pi, 2\pi$

$1 = \tan x \rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}$

Remember  $\frac{\sin x}{\cos x} = \tan x$

$x = 0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}, 2\pi$

$\therefore$  places where  $\sin x = \cos x$   
can = 1 (i.e. at  $\frac{\pi}{4}$ )

$\tan x = 0$  when  $\frac{0}{1}$  is possible. Since  $\tan x = \frac{\sin x}{\cos x}$ ,  $\sin x = 0, \cos x = 1$

$$(8) \quad 2\cos x - \sin x + 2\cos x \sin x = 1$$

$$2\cos x + 2\cos x \sin x - \sin x - 1 = 0$$

$$2\cos x (1 + \sin x) - (\sin x + 1) = 0$$

$$(2\cos x - 1)(\sin x + 1) = 0$$

$$\cos x = \frac{1}{2} \quad x = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\sin x = -1 \quad x = \frac{3\pi}{2}$$

$$x = \frac{\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}$$

$$(9) \quad \cos x = 1 + \sin^2 x$$

$$\cos x = 1 + (1 - \cos^2 x)$$

$$\cos x = 1 + 1 - \cos^2 x$$

$$\cos^2 x + \cos x - 2 = 0$$

$$(\cos x - 1)(\cos x + 2) = 0$$

$$\cos x = 1$$

↓

$$x = 0, 2\pi$$

$$\cos x = -2$$

NO SOLUTION

$$x = 0, 2\pi$$

$$(10) \quad 7\sin x + 5 = 2\cos^2 x$$

$$7\sin x + 5 = 2(1 - \sin^2 x)$$

$$7\sin x + 5 = 2 - 2\sin^2 x$$

$$2\sin^2 x + 7\sin x + 3 = 0$$

$$(2\sin x + 1)(\sin x + 3)$$

$$\sin x = -\frac{1}{2}$$

↓

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\sin x = -3$$

NO

SOLUTION

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

PRACTICE PROBLEMS

①  $1 + \sin x = 2 \cos^2 x$   
 $1 + \sin x = 2(1 - \sin^2 x)$   
 $1 + \sin x = 2 - 2 \sin^2 x$   
 $2 \sin^2 x + \sin x - 1 = 0$   
 $(2 \sin x - 1)(\sin x + 1)$   
 $\sin x = \frac{1}{2} \quad \sin x = -1$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

②  $-3 \cos x + 3 = 2 \sin^2 x$   
 $-3 \cos x + 3 = 2(1 - \cos^2 x)$   
 $-3 \cos x + 3 = 2 - 2 \cos^2 x$   
 $2 \cos^2 x - 3 \cos x + 1 = 0$   
 $(2 \cos x - 1)(\cos x - 1) = 0$   
 $\cos x = \frac{1}{2} \quad \cos x = 1$   
 $x = \frac{\pi}{3}, \frac{5\pi}{3} \quad x = 0, 2\pi$

$x = 0, \frac{\pi}{3}, \frac{5\pi}{3}, 2\pi$

③  $\cos^3 x = \cos^2 x$   
 $\cos^3 x - \cos^2 x = 0$   
 $\cos^2 x (\cos x - 1) = 0$   
 $\cos^2 x = 0 \quad \cos x = 1$   
 $\therefore \cos x = 0$

$x = 0, \frac{\pi}{2}, \frac{3\pi}{2}, 2\pi$

④  $2 \cos^2 x - \cos x = 3$   
 $2 \cos^2 x - \cos x - 3 = 0$   
 $(2 \cos x - 3)(\cos x + 1)$   
 $\cos x = \frac{3}{2} \quad \cos x = -1$   
 NO SOLUTION  $x = \pi$

⑤  $2 \sin^2 x = \cos x + 1$   
 $2(1 - \cos^2 x) = \cos x + 1$   
 $2 - 2 \cos^2 x = \cos x + 1$   
 $0 = 2 \cos^2 x + \cos x - 1$   
 $0 = (2 \cos x - 1)(\cos x + 1)$   
 $\cos x = \frac{1}{2} \quad \cos x = -1$   
 $x = \frac{\pi}{3}, \frac{5\pi}{3} \quad x = \pi$

$x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

⑥  $2 \cos^2 x + 3 \sin x = 3$   
 $2(1 - \sin^2 x) + 3 \sin x = 3$   
 $2 - 2 \sin^2 x + 3 \sin x = 3$   
 $0 = 2 \sin^2 x - 3 \sin x + 1$   
 $0 = (2 \sin x - 1)(\sin x - 1)$   
 $\sin x = \frac{1}{2} \quad \sin x = 1$   
 $x = \frac{\pi}{6}, \frac{5\pi}{6} \quad x = \frac{\pi}{2}$

$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

$$\begin{aligned} \textcircled{7} \quad \sec^2 x &= 4 \\ \tan^2 x + 1 &= 4 \\ \tan^2 x &= 3 \\ \tan^2 x - 3 &= 0 \\ \tan x &= \pm \sqrt{3} \end{aligned}$$

Hint:  $\tan x = \frac{\sin x}{\cos x} \rightarrow \frac{\sqrt{3}}{1} = \frac{1}{\frac{1}{\sqrt{3}}} = \sqrt{3}$

$\therefore$   $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$  ... So wherever this is the case.

$$\begin{aligned} \textcircled{8} \quad \tan x \sin^2 x &= \frac{3}{4} \tan x \\ \tan x \sin^2 x - \frac{3}{4} \tan x &= 0 \\ \tan x \left( \sin^2 x - \frac{3}{4} \right) &= 0 \end{aligned}$$

$$\begin{aligned} \tan x = 0 & \quad \sin^2 x = \frac{3}{4} \\ \downarrow & \quad \sin x = \pm \frac{\sqrt{3}}{2} \\ x = 0, \pi & \quad x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \end{aligned}$$

$$x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$$

$$\textcircled{9} \quad \frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x} = 4$$

$$\frac{(1 + \sin x)^2 + \cos^2 x}{\cos x (1 + \sin x)} = 4$$

$$1 + 2\sin x + \sin^2 x + \cos^2 x = 4\cos x (1 + \sin x)$$

$$1 + 2\sin x + 1 = 4\cos x + 4\cos x \sin x$$

$$2 + 2\sin x = 4\cos x (1 + \sin x)$$

$$2(1 + \sin x) = 4\cos x (1 + \sin x)$$

$$2 = 4\cos x$$

$$0 = 4\cos x - 2$$

$$0 = 2(2\cos x - 1)$$

$$\cos x = \frac{1}{2}$$

$$\begin{aligned} \textcircled{10} \quad \sin^2 x - \frac{1}{2} \cos x + \cos x \sin^2 x &= \frac{1}{2} \\ \sin^2 x + \cos x \sin^2 x &= \frac{1}{2} + \frac{1}{2} \cos x \\ \sin^2 x (1 + \cos x) &= \frac{1}{2} (1 + \cos x) \\ \sin^2 x &= \frac{1}{2} \end{aligned}$$

$$\sin x = \pm \frac{1}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$\textcircled{11} \quad \cot x = \cos x$$

$$\frac{\cos x}{\sin x} = \cos x$$

$$\cos x = \cos x \sin x$$

$$0 = \cos x \sin x - \cos x$$

$$0 = \cos x (\sin x - 1)$$

$$\cos x = 0 \quad \sin x = 1$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2} \quad x = \frac{\pi}{2}$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\textcircled{12} \quad \tan x - \sqrt{2} = \frac{1}{\tan x + \sqrt{2}}$$

$$\tan^2 x - 2 = 1$$

$$\tan^2 x = 3$$

$$\tan x = \pm \sqrt{3} \quad (\text{see } \textcircled{7})$$

$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$