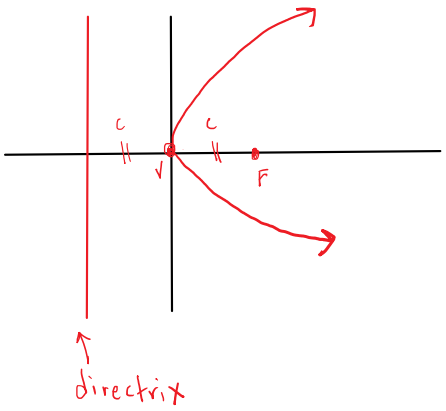
Parabolas

April 21, 2022 2:52 PM

- locus of points equidistant from a fixed point (focus) and a line (directrix)

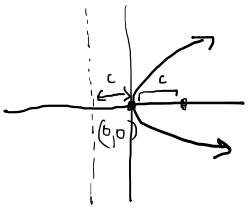


- vertex is $\frac{1}{2}$ way between directrix & focus
- the axis of symmetry is $\perp$ to the directrix

$y = x^2$

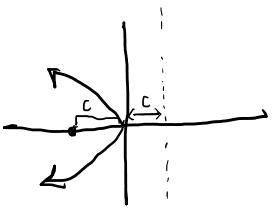
Rules

h,k = vertex



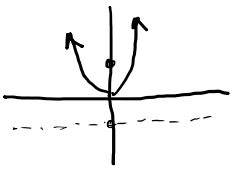
- opens right
- Focus $(h+c, 0)$

$y^2 = 4cx$ or transformed $(y-k)^2 = 4c(x-h)$



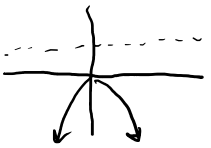
- opens left

$\Rightarrow y^2 = -4cx$ or $(y-k)^2 = -4c(x-h)$



$x^2 = 4cy$

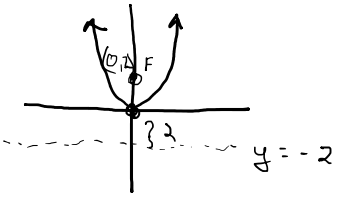
$(x-h)^2 = 4c(y-k)$



$x^2 = -4cy$

$(x-h)^2 = -4c(y-k)$

ex. Sketch $x^2 = 8y$ $\quad (x-h)^2 = 4c(y-k)$



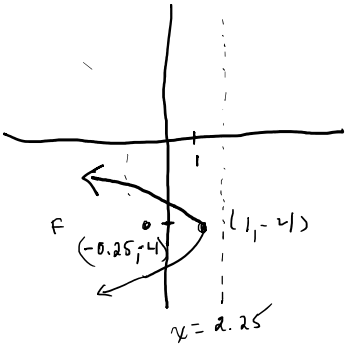
$4c = 8$

$c = 2$

$x^2 = 8y$

$x^2 = 4cy$

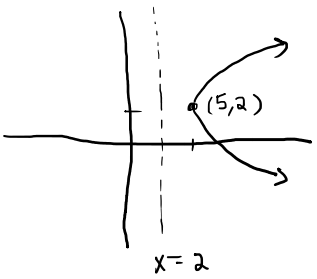
ex. Sketch $(y+4)^2 = -5(x-1)$ $\longrightarrow$



$-4c = -5$

$c = \frac{5}{4}$ or 1.25

ex. Find the rule:

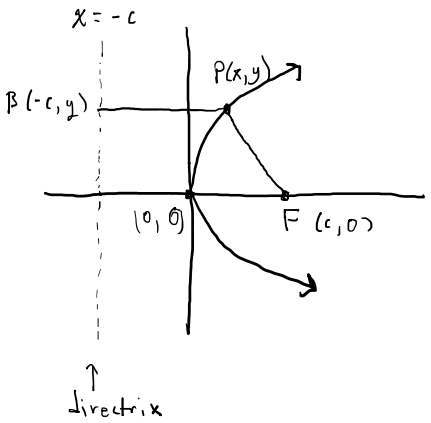


$(y-k)^2 = 4c(x-h)$

$(y-2)^2 = 4(3)(x-5)$

$(y-2)^2 = 12(x-5)$

Derivation of the formula:



$PF = PB \quad \ldots (i)$

By the distance formula,

$PF = \sqrt{(x-c)^2 + y^2}$

$PB = \sqrt{(x+c)^2}$

Since $PF = PB$ [from (i)], we get

$\sqrt{(x-c)^2 + y^2} = \sqrt{(x+c)^2}$

by squaring both sides

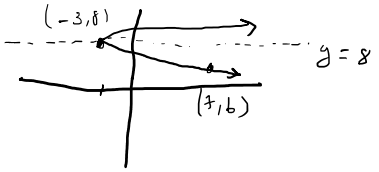
$(x-c)^2 + y^2 = (x+c)^2$

$x^2 - 2cx + c^2 + y^2 = x^2 + 2cx + c^2$

further, by solving

$y^2 = 4cx$

Ex. A parabola passes through (7,6) and has an axis of symmetry of $y=8$ and a vertex at (-3,8). Find the rule.



$(y-k)^2 = 4c(x-h)$

$(y-8)^2 = 4c(x+3)$

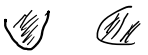
$(6-8)^2 = 4c(7+3)$

$4 = 4c(10)$

$0.1 = c$

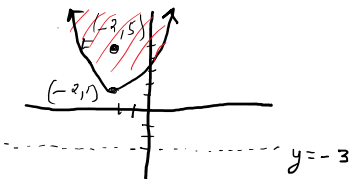
$(y-8)^2 = 0.4(x+3)$

Inequalities / shading: If $x^2 \leq / <$ or $y^2 \leq / <$ } shade inside



If $x^2 > \text{ or } \geq$ or $y^2 \geq \text{ or } >$ } shade outside

ex. Find the inequality of parabola with a focus at (-2,5) and a directrix of $y=-3$, if the focus & vertex are part of the solution set.



$V = (-2,1)$

$c = 4$

$(x-h)^2 = 4c(y-k)$ ← sign?

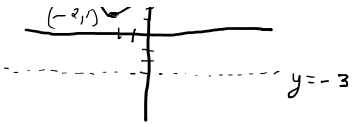
$(x+2)^2 = 16(y-1)$

$\leq$

$(x+2)^2 \leq 16(y-1)$

test point: (-2,6)

$(-2+2)^2 \leq 16(6-1)$



$(x-h)^2 = 4c(y-k)$

$(x+2)^2 = 16(y-1)$

test point: $(-2,6)$

$(-2+2)^2 \leq 16(6-1)$

$0 \leq 80$ ✓ True

Point of intersection

- use any method: substitution, elimination, comparison

ex. Solve for point(s) of intersection between $y = -12x + 40$ and $y = 4(x-4)^2$

$-12x + 40 = 4(x-4)^2$

$-3x + 10 = (x-4)^2$

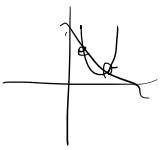
$-3x + 10 = x^2 - 8x + 16$

$0 = x^2 - 5x + 6$

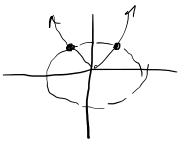
$0 = (x-2)(x-3)$

$P_1 = (2, 16)$

$P_2 = (3, 4)$



ex. $x^2 = 28.125y$ and $\frac{x^2}{625} + \frac{y^2}{100} = 1$



$100x^2 + 625y^2 = 62500$

$100(28.125y) + 625y^2 = 62500$

$625y^2 + 2812.5y - 62500 = 0$

Use quadratic formula $a = 625$, $b = 2812.5$, $c = -62500$

$y_1 = 8$ ✓

$y_2 = 12.5$ ← this is extraneous

$x^2 = 28.125(8)$

$x^2 = 225$

$x_1 = 15$

$x_2 = -15$

$P_1 = (15, 8)$

$P_2 = (-15, 8)$

Eccentricity

$\frac{c}{a}$ { for ellipses & hyperbolas

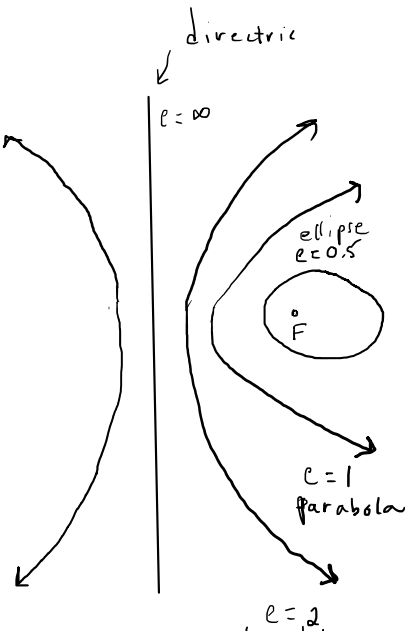
- Tells you how "un-circular" the curve is
- At eccentricity = 0 we get a circle

$0 <$ eccentricity $(e) < 1$ = ellipse

$= 1$ we get a parabola

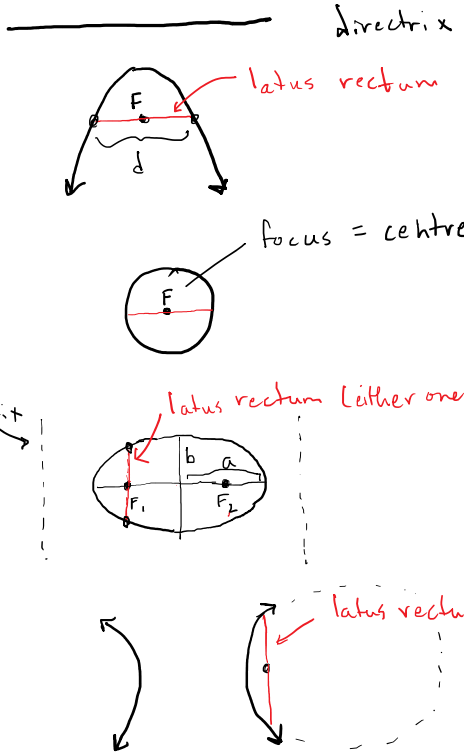
> 1 we get a hyperbola

infinite eccentricity = line



Latus Rectum

- not a body part
- runs parallel to the directrix and passes through the focus



Parabola: 4x the focal length

Circles: it is the diameter

Ellipses: $\frac{2b^2}{a}$

Hyperbola: $\frac{2b^2}{a}$