

# Graph Theory

## Definitions:

Order of a graph: number of vertices it has

Degree of a vertex: how many edges touch it.

Circuit: a path that begins and ends at same vertex

"Simple" circuit: does not repeat any edges

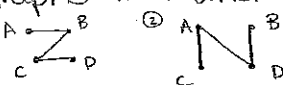
Hamiltonian path: travels over (to) every VERTEX once and only once

Euler path: travels over every EDGE once and only once.

\* if one exists \* only 2 vertices of odd degree.

Tree Diagram: a connected graph with no simple circuits

Complementary Graphs: two graphs that when overlapped create a COMPLETE graph



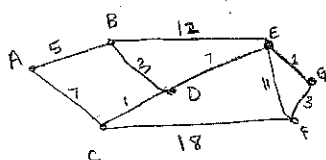
Digraph: a.k.a. DIRECTED GRAPH

edges have arrowheads to indicate "one way streets"

Network: edges have weights to indicate value of edge (cost, ...)

## Processes:

① Path of Optimal Value: a tree diagram!  
NO START GIVEN, FIND MINIMUM IN EXAMPLE

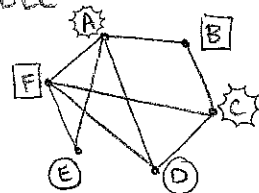


- HIGHLIGHT EDGE OF LOWEST VALUE (CD)
- HIGHLIGHT NEXT LOWEST (EQ) DOES NOT HAVE TO BE ADJACENT
- CONTINUE UNTIL ALL VERTICES ARE CONNECTED WITHOUT CREATING ANY SIMPLE CIRCUITS (GF AND ED), (AB), (ED)

② Chromatic Number: To sort items that are "INCOMPATIBLE"

VERTEX	INCOMPATIBLE
A	B, D, E, F
B	A, C
C	B, D, F
D	A, C, F
E	A, F
F	A, C, D, E

- DRAW VERTICES IN CIRCULAR FASHION
- CONNECT VERTICES THAT ARE "INCOMPATIBLE"
- COLOR A VERTEX OF HIGHEST DEGREE (A)
- COLOR AN ADJACENT VERTEX A DIFFERENT COLOR (F) UNTIL ALL VERTICES ARE COLOURED, REPEATING ASAP.
- NUMBER OF COLOURS = CHROMATIC NUMBER (3)

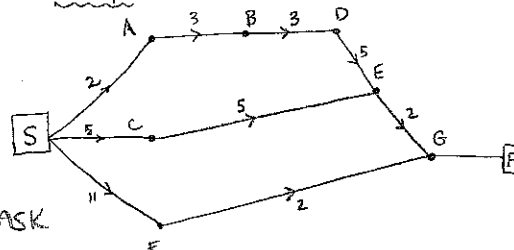


③ Critical Path: "PRIOR STEPS"

TASK	TIME	PRIOR
A	2	NONE
B	3	A
C	5	NONE
D	3	B
E	5	C, D
F	11	NONE
G	2	E, F

- CREATE A START [S]
- READING "PRIOR COLUMN" CONNECT ANY TASKS THAT HAVE NONE TO START, WEIGHT EDGE BEFORE WITH TIME FOR TASK
- CONTINUE READING "PRIORS" CONNECTING VERTICES TO THOSE WHO NEED TO BE COMPLETED FIRST
- TAKE TO FINISH [F]
- LIST ALL POSSIBLE PATH AND FIND VALUE
- CRITICAL PATH = LONGEST ONE

example:



- [S]A[B]D[E]G[F] = 15 ← \*CP\*
- [S]C[E]G[F] = 12
- [S]F[G] = 13

\*NOTE\* edges [E] has NO value!!