

Inverse of a Function

Below are some functions we'll be looking at in this course.

- Absolute value : $f(x) = |x| \rightarrow$ Transformed : $f(x) = a/b|x-h| + k$
- Square root : $f(x) = \sqrt{x} \rightarrow$ " : $f(x) = a\sqrt{b(x-h)} + k$
- rational : $f(x) = \frac{1}{x} \rightarrow$ " : $f(x) = \frac{a}{b(x-h)} + k$

a : vertical stretch or compression and vertical reflection (over the x)

b : vertical stretch or compression and horizontal reflection (over the y)

h : Horizontal shift

k : vertical shift

Inverse of a Function

Swapping of x and y coordinates of the ordered pairs of any function or relation gives its inverse.

If $f(x)$ is the function, then $f^{-1}(x)$ is its inverse.

Ex: $\mathbb{R} = \{(0,1), (2,3), (5,4)\}$ } When graphed, the inverse is a
 $\mathbb{R}^{-1} = \{(1,0), (3,2), (4,5)\}$ } reflection over the line $y=x$ ~~Y~~

The domain of the function becomes the range of its inverse.
 " range " " " " domain " "

Note: The inverse of a function may be a relation or a

To solve for $f^{-1}(x)$, swap x and y, and then isolate y

Given $f(x) = 2x - 5$, find $f^{-1}(x)$:

$$y = 2x - 5$$

$$x = 2y - 5$$

$$x + 5 = 2y$$

$$\frac{x}{2} + \frac{5}{2} = y$$

$$y = \frac{x}{2} + \frac{5}{2}$$

$$f^{-1}(x) = \frac{x}{2} + \frac{5}{2}$$

Given $f(x) = 2(x-6)^2 + 4$, find $f^{-1}(x)$

$$\begin{aligned}
 & y = 2(x-6)^2 + 4 \\
 \text{Swap } l \rightarrow & x = 2(y-6)^2 + 4 \\
 \text{isolate "new" } y \rightarrow & x - 4 = 2(y-6)^2 \\
 & \sqrt{\frac{x}{2} - 2} = \sqrt{(y-6)^2} \\
 & \sqrt{\frac{x}{2} - 2} = y - 6 \\
 & \sqrt{\frac{x}{2} - 2} + 6 = y
 \end{aligned}$$

$$f^{-1}(x) = \sqrt{\frac{x}{2} - 2} + 6$$

Given $f(x) = \frac{4}{3(x-2)} + 1$, find $f^{-1}(x)$

$$y = \frac{4}{3(x-2)} + 1$$

$$x = \frac{4}{3(y-2)} + 1$$

$$\begin{aligned}
 x-1 &= \frac{4}{3(y-2)} \\
 (x-1)(3)(y-2) &= 4 \\
 (3x-3)(y-2) &= 4
 \end{aligned}$$

$$y-2 = \frac{4}{3x-3}$$

$$y = \frac{4}{3x-3} + 2$$

$$f^{-1}(x) = \frac{4}{3x-3} + 2$$

Bonus Given: $f(x) = \frac{x+3}{x+4}$, what is $f^{-1}(x)$?

Operations and Composites

Operations: addition, subtraction, multiplication, division - can be performed on functions.
ex. we can add two functions to make a new function

Addition: $(f+g)(x) = f(x) + g(x)$

Subtraction: $(f-g)(x) = f(x) - g(x)$

Multiplication: $(f \cdot g)(x) = f(x) \cdot g(x)$

Division: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$

Examples: If $h(x) = 4x^2 + 7x$ and $j(x) = 5x - 1$, find:

Examples: If $h(x) = 4x^2 + 7x$ and $j(x) = 5x - 1$, find:

a) $h + j$

$$(4x^2 + 7x) + (5x - 1)$$

$$\rightarrow 4x^2 + 12x - 1$$

b) $h - j$

$$(4x^2 + 7x) - (5x - 1)$$

$$4x^2 + 7x - 5x + 1$$

$$4x^2 + 2x + 1$$

c) $h \cdot j$

$$(4x^2 + 7x)(5x - 1)$$

$$20x^3 - 4x^2 + 35x^2 - 7x$$

$$20x^3 + 31x^2 - 7x$$

d) $\frac{h}{j}$

$$\frac{4x^2 + 7x}{5x - 1}, x \neq \frac{1}{5}$$

Restriction: set the denominator to zero

$$5x - 1 = 0$$

$$5x = 1$$

$$x = \frac{1}{5}$$

Composites

- Where one function is applied to another function

Ex. $f(g(x))$ or $f \circ g$ or "f of g of x" or "f after g"

$g(f(x))$ or $g \circ f$ or "g of f of x" or "g after f"

Given: $f(x) = 2x + 1$ and $g(x) = x^2 - 2x$, find:

$$h(x) = f(g(x))$$

$2x + 1$ start with $f(x)$

$$2(x^2 - 2x) + 1 \quad \text{replace } x \text{ with } g(x)$$

simplify

$$h(x) = 2x^2 - 4x + 1$$

$$h(x) = g(f(x))$$

$x^2 - 2x$ start with $g(x)$

$$(2x+1)^2 - 2(2x+1) \quad \text{replace } x \text{ with } f(x)$$

$$4x^2 + 4x + 1 - 4x - 2 \quad \text{simplify}$$

$$h(x) = 4x^2 - 1$$

NON COMMUTATIVE!
 $f \circ g \neq g \circ f$

$$\begin{aligned} h(-2) &= f(g(-2)) \\ f(g(x)) &= 2x^2 - 4x + 1 \\ h(x) &= f(g(-2)) = 2(-2)^2 - 4(-2) + 1 \\ h(x) &= 17 \end{aligned}$$

Alternate

$$h(x) = f(g(-2))$$

$$g(-2) = (-2)^2 - 2(-2)$$

$$g(-2) = 8$$

$$f(8) = 2(8) + 1 \quad (f(x) = 2x + 1)$$

Example: In words

You are gift shopping for a friend's birthday. A store has a "half-off" sale on shirts, and you want to buy your friend three of them. Also, from the same store, you get your friend a \$10 gift certificate.

① What are the functions? (Independently) $f(x) = 0.5x$

② What is the composite function? $(g \circ f)(x) = 3(0.5x) + 10$

③ Original cost of the shirt is \$25,
how much did you spend on \rightarrow your shopping trip.

$$(g \circ f)(x) = 3(0.5(25)) + 10 \\ = \$\underline{47.50}$$