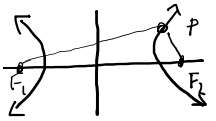
Hyperbolas

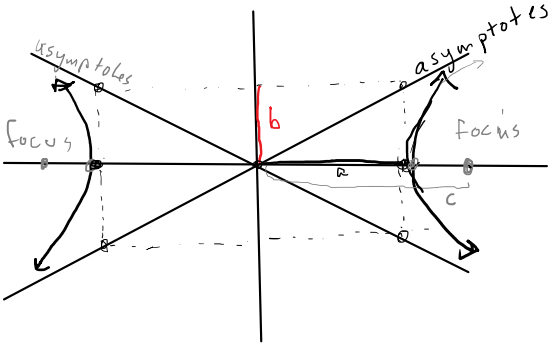
April 6, 2022 11:03 AM

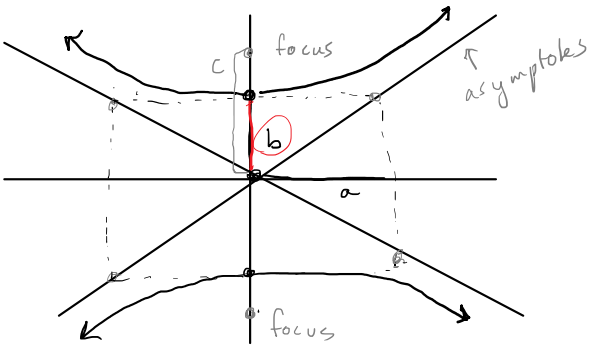
- A hyperbola is a locus of points whereby the absolute value of the difference from the foci to a point is always the same.

$$|PF_1 - PF_2| = |P_2F_1 - P_2F_2|$$



$$* \quad |PF_1 - PF_2| = 2a$$





- the centre of the hyperbola is halfway between the foci, or similarly, the halfway point between the vertices

Rule:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

opens horizontally

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

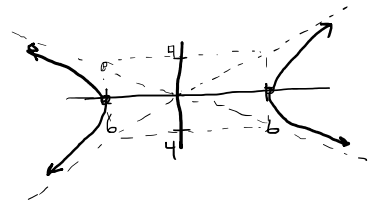
opens vertically

- To find the focal length, $\boxed{c^2 = a^2 + b^2}$
- The a and b will determine the equations of asymptotes.

$$y = \pm \frac{b}{a}x \quad \leftarrow \text{at the origin}$$

$$y = \pm \frac{b}{a}(x-h) + k$$

Ex. Sketch $\frac{x^2}{36} - \frac{y^2}{16} = 1$ ← horizontal



Sketch: $\frac{(x-2)^2}{25} - \frac{(y+4)^2}{49} = -1$

Writing in Standard Form ... keep on completing the square

ex. $4x^2 - y^2 - 16x - 4y - 4 = 0$

ex. $4x^2 - y^2 - 16x - 4y - 4 = 0$

$(4x^2 - 16x\ \ ) - (y^2 + 4y\ ) = 4$

$4(x^2 - 4x + 4) - (y^2 + 4y + 4) = 4 + 16 - 4$

$4(x-2)^2 - (y+2)^2 = 16$

$\frac{4(x-2)^2}{16} - \frac{(y+2)^2}{16} = \frac{16}{16}$

$\frac{(x-2)^2}{4} - \frac{(y+2)^2}{16} = 1$

ex. $-9x^2 + 16y^2 + 54x + 64y - 161 = 0$

$-9x^2 + 54x + 16y^2 + 64y = 161$

$-9(x^2 - 6x + 9) + 16(y^2 + 4y + 4) = 161 - 81 + 64$

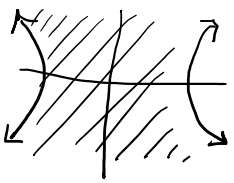
$-9(x-3)^2 + 16(y+2)^2 = 144$

$\frac{-9(x-3)^2}{-144} + \frac{16(y+2)^2}{-144} = \frac{144}{-144}$

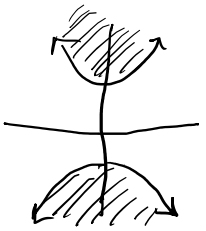
$\frac{(x-3)^2}{16} - \frac{(y+2)^2}{9} = -1$

Hyperbola: Inequalities

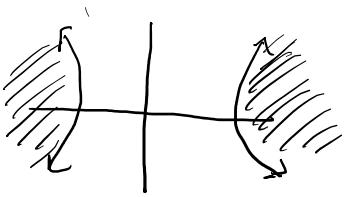
If ≤ 1



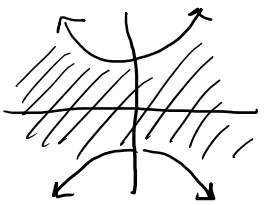
≤ -1



≥ 1



≥ -1



$\frac{x^2}{a^2} - \frac{y^2}{b^2}$ ≤ 1 (≥ 1) ≤ -1 ≥ -1

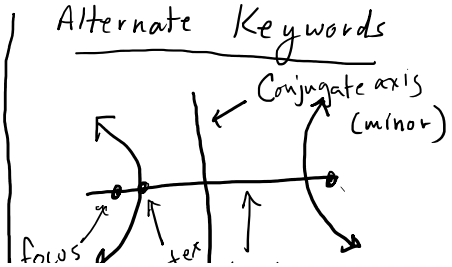
Sketch: $\frac{(x-1)^2}{9} - \frac{(y-2)^2}{16} \leq -1$

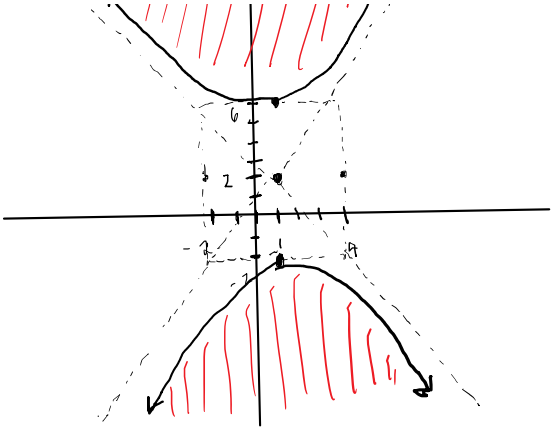
Centre (1, 2)

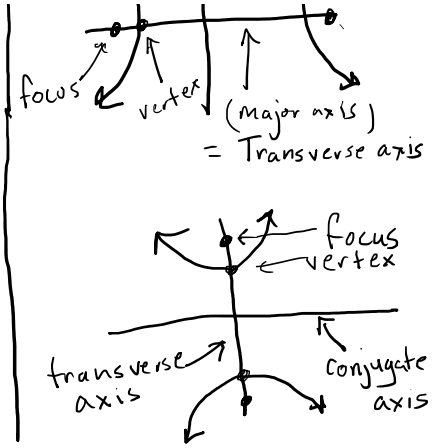
x direction = 3

y direction = 4

Alternate Keywords







Ex. A hyperbola is centred at (2,3) and the coordinates of one vertex are at (4,3). If the focal length is $\sqrt{13}$, find the inequality given $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} < 1$

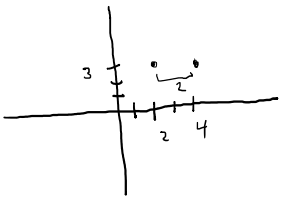
$c^2 = a^2 + b^2$

$(\sqrt{13})^2 = a^2 + b^2$

$13 = 4 + b^2$

$9 = b^2$

$3 = b$



$\frac{(x-2)^2}{4} - \frac{(y-3)^2}{9} < 1$

Point of intersection

- use substitution, comparison & elimination → follow algebra → check for extraneous root

Solve for point(s) of intersection between $\frac{x^2}{25} - \frac{y^2}{64} = 1$ and $\left(\frac{9}{9}\right)\frac{(x-4)^2}{9} + \frac{y^2}{81} = 1$

$\frac{x^2}{25} - \frac{y^2}{64} = 1$

$* \quad 64x^2 - 25y^2 = 1600$

$\frac{9(x-4)^2}{81} + \frac{y^2}{81} = 1$

$* \quad 9(x-4)^2 + y^2 = 81$

$y^2 = -9(x-4)^2 + 81$

$64x^2 - 25(-9(x-4)^2 + 81) = 1600$

$64x^2 - 25(-9(x-4)(x-4) + 81) = 1600$

$64x^2 - 25(-9(x^2 - 8x + 16) + 81) = 1600$

$64x^2 - 25(-9x^2 + 72x - 144 + 81) = 1600$ (−63)

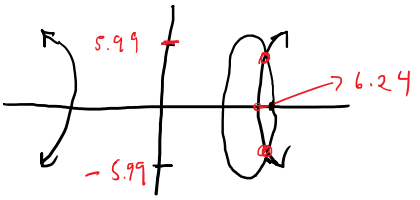
$64x^2 + 225x^2 - 1800x + 1575 = 1600$

$289x^2 - 1800x - 25 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = -0.014$ (extraneous) → $y^2 = -9(-0.014 - 4)^2 + 81 = -64.00$ (can't square root this) NO SOLUTION

$x = 6.24$ → $y^2 = -9(6.24 - 4)^2 + 81 \quad y = \pm 5.99$



(6.24, 5.99) ✓

(6.24, -5.99) ✓