# FINDING THE RULE

Exponential Functions

**Miss Connell**

# < 4 Parameters > to < 2 Parameters >

- $f(x) = ac^{b(x-h)} + k$ < 4 Parameters >
- $f(x) = a(c^b)^{x-h} + k$
- $f(x) = ac^{x-h} + k$ < 3 Parameters >
- $f(x) = a\left(\frac{c^x}{c^h}\right) + k$ Can also be written as…
- $f(x) = \left(\frac{a}{c^h}\right)c^x + k$ We have a new “a” value!
- $f(x) = ac^x + k$ < 2 Parameters >

# Converting

- When finding the rule

# There are 3 situations…

- $f(x) = ac^x + k$

- Where k is equal to zero!

$$f(x) = ac^x$$

# First

- $f(x) = ac^x + k$
- Where $c = (1 - r)$ or $c = (1 + r)$
- DECAY and GROWTH

# Second

- Compound Interest/Growth

# Third

# Finding the Rule

## With Form $f(x) = ac^x$

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- **Case One:** Initial Value is Given

- Find the rule of an exponential curve passing by $(0, -2)$ and $(2, -8)$

- ***Step One***: Find $a \Rightarrow use\ the\ initial\ value$!

$$-2 = ac^0$$
$$-2 = a(1)$$
$$-2 = a$$

# Finding the Rule

## With Form $f(x) = ac^x$

- **<u>Case One:</u>** Initial Value is Given
- Exponential curve passing by $(0, -2)$ and $(2, -8)$
- ***<u>Step Two</u>***: Find $c \Rightarrow$ *substitute the point in*!

$$(2, -8) \mapsto f(x) = ac^x \quad \begin{matrix} c > 0 \\ c \neq 1 \end{matrix}$$

$$f(x) = -2c^x$$

$$-8 = -2c^2$$

$$4 = c^2$$

$$\sqrt{4} = \sqrt{c^2}$$

$$\pm 2 = c$$

$$2 = c$$

$$\therefore f(x) = -2(2)^x$$

- **Case Two**: "Two Ordered Pairs"
- Find the rule of an exponential curve passing by $(1, 360)$ and $(4, 45\,000)$

- ***Step One***: Write two equations with two unknowns

$$f_1(x) = ac^x \mapsto 360 = ac^1$$
$$f_2(x) = ac^x \mapsto 45000 = ac^4$$

# Finding the Rule

## With Form $f(x) = ac^x$

- <u>**Case Two**</u>**:** "Two Ordered Pairs"
- Exponential curve passing $(1, 360)$ & $(4, 45\,000)$
- <u>***Step Two***</u>: Use the method of substitution to solve

Rearrange (2) to isolate $a$

$$45000 = ac^4$$

$$\frac{45000}{c^4} = a$$

Substitute (2) into (1) {mathematical notation $(2) \mapsto (1)$}

$$360 = ac^1$$

$$360 = \left(\frac{45000}{c^4}\right)c^1$$

- **Case Two/*Step Two***: Solve

$$360 = \left(\frac{45000}{c^4}\right)c^1$$

$$360 = \left(\frac{45000c^1}{c^4}\right)$$

$$\frac{c^1}{c^4} \rightarrow 1 - 4 = -3 \rightarrow \frac{1}{c^3}$$

$$360 = \frac{45000}{c^3}$$

$$c^3 = \frac{45000}{360}$$

$$c^3 = 125$$

$$c = 5$$

- **<u>Case Two</u>:** "Two Ordered Pairs"
- Find the rule of an exponential curve passing by $(1, 360)$ and $(4, 45\,000)$

- ***<u>Step Three</u>***: Find '$a$' by substituting in $c = 5$ and a point

$$c = 5 \mapsto f(x) = ac^x$$
$$(1, 360) \mapsto f(x) = a(5)^x$$

$$360 = a(5)^1$$
$$360 = a(5)$$
$$72 = a$$

$$\therefore f(x) = 72(5)^x$$

- The number of people living in Kilwat, Germany, varies according to the rule of an exponential function. On January 1st 1975, the city's population was 130 000. One January 1st 1985, it was 260 000.

- What was the population of this German city on January 1st 2000, given that the growth rate remained constant?

- Show all your work!!

# Try This!

- Key information:
- ~~The number of people living in Kilwat, Germany, varies according to the rule of~~ an exponential function. ~~On January 1st~~ 1975~~, the city's population was~~ 130 000. ~~On January 1st~~ 1985, ~~it was~~ 260 000.

This is like Case 1 of the previous examples! ☺

$f(x) = ac^x$, where $f$ is population, $x$ is time (years)

January 1st 1975 → YEAR 0 → $x = 0, f(0) = 130\ 000$

January 1st 1985 → YEAR 10 → $x = 10, f(10) = 260\ 000$

January 1st 2000 → YEAR 25 → $x = 25, f(25) = ?$

# Try This!

- **Case One**: Initial Value is Given

- Find the rule of an exponential curve passing by $(0, 130\,000)$ and $(10, 260\,000)$

- *Step One*: Find $a \Rightarrow use\ the\ initial\ value$!

$$130\,000 = ac^0$$
$$130\,000 = a(1)$$
$$130\,000 = a$$

# Finding the Rule

## With Form $f(x) = ac^x$

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- Find the rule of an exponential curve passing by $(0, 130\,000)$ and $(10, 260\,000)$

- *Step Two*: Find $c \Rightarrow$ *substitute the point in*!

$$(10, 260\,000) \mapsto f(x) = ac^x$$
$$f(x) = 130\,000\,c^x$$
$$260\,000 = 130\,000\,c^{10}$$
$$2 = c^{10}$$
$$\sqrt[10]{2} = \sqrt[10]{c^{10}}$$
$$2^{\frac{1}{10}} = c$$

- $f(x) = 130\,000\left(2^{\frac{1}{10}}\right)^x$

- $\therefore$ the rule of the function is $f(x) = 130\,000(2)^{\frac{x}{10}}$

- The rule is $f(x) = 130\,000\left(2^{\frac{1}{10}}\right)^{x}$, now find the population at YEAR 25.

$$f(25) = 130\,000(2)^{\frac{25}{10}}$$

$$f(25) \approx 735\,391$$

∴the population of Kilwat on January 1st 2000 was ≈ 735 391

- These are growth and decay problems.
- $f(x) = ac^x$
- where $a = initial\ value$ and
- $c = base$ and the base is represented by:
  - $c = (1 - r)$ for decay process
  - $c = (1 + r)$ for growth process
    - where $r = growth\ or\ decay\ rate$

# Finding the Rule

## With Form $f(x) = a(1 \pm r)^x$

Ex. A ball which is 2m above a table is dropped and subsequent heights are measured. Students notice that the ball loses 20% of its height after each bounce.

- $f(x) = ac^x$
- Initial Value? 2
- Growth/Decay?
- $(1 - r)$
- Rate? 20% $\rightarrow$ 0.2

$$f(x) = 2(1 - 0.20)^x$$

$$f(x) = 2(0.80)^x$$

The height of each bounce is 80% of the height of the previous bounce.

Number of bounces

- To cover the cost of building a water filtration plant, a municipality is planning an average tax increase of 6% per year starting in 1994.
- Mr. Blais paid $1225 in taxes for the year 1993. He wants to find a function, *t*, that can be used to calculate the amount of annual taxes as a function of the number of years, *n*, elapsed since 1993.
- What rule corresponds to function *t*?

# Try This!

- Key Information:
- To cover the cost of building a water filtration plant, a municipality is planning an average tax increase of 6% per year starting in 1994.

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# Try This!

Initial Value

Growth or Decay Rate

- Key Information:
- To cover the cost of building a water filtration plant, a municipality is planning an average tax increase of 6% per year starting in 1994.
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$$t(n) = a(1 + r)^n$$

# Try This!

Initial Value

Growth or Decay Rate

- Key Information:
- To cover the cost of building a water filtration plant, a municipality is planning an average tax increase of 6% per year starting in 1994.
- Mr. Blais paid $1225 in taxes for the year 1993. He wants to find a function, $t$, that can be used to calculate the amount of annual taxes as a function of the number of years, $n$,

$$t(n) = a(1 + r)^n$$

# Try This!

$$t(n) = a(1 + 0.06)^n$$

$$\boldsymbol{Check\ Initial\ Value:}$$

$$\boldsymbol{t = tax\ amount\ (\$)}$$

$$\boldsymbol{n = number\ of\ yrs\ since\ 1993}$$

$$\boldsymbol{a = initial\ value}$$

$$\boldsymbol{t(n) = a(1.06)^n}$$

$$\boldsymbol{1993\ (YEAR\ 0)}$$

$$\boldsymbol{n = 0 \rightarrow t(0) = \$1225}$$

$$\boldsymbol{1225 = a(1.06)^0}$$

$$\boldsymbol{1225 = a(1)}$$

$$\boldsymbol{a = 1225}$$

- Key Information:
- To cover the cost of building a water filtration plant, a municipality is planning an average tax increase of 6% per year starting in 1994.

$$t(n) = 1225(1.06)^n$$

- Mr. Bluis paid $1225 in taxes for the year 1993. He wants to find a function, *t*, that can be used to calculate the amount of annual taxes as a function of the number of years, *n*, elapsed since 1993.

- In 1987 the world population reached 5 billion, and was increasing at a rate of approximately 1.6% per year.
- If this rate of growth is maintained, what would be the world population in 2037?

# Growth of Populations

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- In 1987 the world population reached 5 billion, and was increasing at a rate of approximately 1.6% per year.
- If this rate of growth is maintained, what would be the world population in 2037?

$$f(x) = a(1 + r)^x$$

$f$: *population* (in billions)
$x$: *number of year since* 1987
$a$: *initial value*

# Growth of Populations

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YEAR 0

Initial value

- In 1987 the world population reached 5 billion, and was increasing at a rate of approximately 1.6% per year.

Rate → 0.016

- If this rate of growth is maintained, what would be the world population in 2037?

$$f(x) = 5(1 + 0.016)^x$$

$$f(x) = 5(1.016)^x$$

# Growth of Populations

$$f(x) = 5(1.016)^x$$

- In 1987 the world population reached 5 billion, and was increasing at a rate of approximately 1.6% per year.

1987 to 2037 is 50 years

$$f(50) = 5(1.016)^{50}$$

- If this rate of growth is maintained, what would be the world population in 2037?

$$f(50) \approx 11.06$$

$$\therefore \textit{the population in } 2037 \textit{ would be approximately } 11 \textit{ Billion}$$

# Growth of Populations

# Compound Interest Problems

- $\rightarrow annually$: $y = a(1+r)^x$
- $\rightarrow semi-annually$: $y = a\left(1+\frac{r}{2}\right)^{2x}$
- $\rightarrow quarterly$: $y = a\left(1+\frac{r}{4}\right)^{4x}$
- $\rightarrow monthly$: $y = a\left(1+\frac{r}{12}\right)^{12x}$
- $\rightarrow daily$: $y = a\left(1+\frac{r}{365}\right)^{365x}$
- $\rightarrow every\ 5\ years$: $y = a(1+r)^{\frac{x}{5}}$

- General Rule:

$$c_x = c_o\left(1+\frac{r}{k}\right)^{kx}$$

Where:

$x: number\ of\ years$

$k: number\ of\ time\ per\ year\ that\ interest\ is\ paid$

$r: annual\ interest\ rate$

$c_o: capital\ invested$

$c_x: capital\ after\ 'x'\ years$

# Compound Interest

- Ex: After studying the evolution of a population of 2000 orcs, Gandalf concluded that the population increased by 15% every two years.

- What rule can be used to find the number, $o$, of orcs there will be in $t$ years?

# Compound Growth

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- Ex: After studying the evolution of a population of 2000 orcs, Gandalf concluded that the population increased by 15% every two years.

- What rule can be used to find the number, $o$, of orcs there will be in $t$ years?

$$Initial\ Value: 2000$$

$$o(t) = 2000(1 + 0.15)^{\frac{t}{2}}$$

$$o(t) = 2000(1.15)^{\frac{t}{2}}$$

# Compound Growth

- Three years ago Greg invested \$1000 at a fixed interest rate compounded every 6 months. His investment is currently valued at \$1400.
- Given $c_n = c_o\left(1 + \frac{t}{k}\right)^{nk}$

Where:

$n$: *number of years*

$k$: *number of times per year that interest is paid*

$t$: *annual interest rate*

$c_o$: *capital invested*

$c_n$: *capital after 'n' years*

- To the nearest hundredth, what is the annual rate of interest?
- Show your work.

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# Try This!

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Where:

$n$: *number of years*
$k$: *number of times per year that interest is paid*
$t$: *annual interest rate*
$c_o$: *capital invested*
$c_n$: *capital after 'n' years*

Every 6 months – semi-annually: $(k = 2)$

$$c_n = c_o\left(1 + \frac{t}{2}\right)^{2n}$$

$$1400 = 1000\left(1 + \frac{t}{2}\right)^{2(3)}$$

- Three years ago Greg invested $1000 at a fixed interest rate compounded every 6 months. His investment is currently valued at $1400.

- Given $c_n = c_o\left(1+\frac{t}{k}\right)^{nk}$

Where:

$n$: *number of years*
$k$: *number of times per year that interest is paid*
$t$: *annual interest rate*
$c_o$: *capital invested*
$c_n$: *capital after 'n' years*

$$1400 = 1000\left(1+\frac{t}{2}\right)^{2(3)}$$

$$1.4 = \left(1+\frac{t}{2}\right)^{6}$$

$$\sqrt[6]{1.4} = \sqrt[6]{\left(1+\frac{t}{2}\right)^{6}}$$

$$1.4^{\frac{1}{6}} = 1+\frac{t}{2}$$

$$1.05768 \approx 1+\frac{t}{2}$$

$$0.05768 \approx \frac{t}{2}$$

$$0.1154 \approx t$$

$\therefore$ *The annual rate is* $0.1154$ *or* $11.54\%$

# Try This!

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# 11, 12, 13, 14, 15ab, 17

Using first type!

Using $(1 \pm r)$ type!

# Homework