Absolute Value Functions

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Case 1

1) Write the general form of the rule
2) Substitute in (h,k), given the vertex
3) Substitute in (x,y), given the additional point
4) Solve for a
5) Write the rule (replacing a,h,k)

Example: Determine the rule of an absolute value function with a vertex $(3,5)$ and passes through $(5,8)$.
x y

$f(x) = a|(x-h)| + k$

$f(x) = a|(x-3)| + 5$

$8 = a|(5-3)| + 5$

$8 = a|(2)| + 5$

$3 = 2a$

$\frac{3}{2} = a$

$f(x) = \frac{3}{2}|(x-3)| + 5$

Case 2: If we've given a vertex, without it being labelled as such, we can find the rule by:

1) Using the two points with the same y-value to determine h.
2) Check that the point given is actually the vertex (does $x = h$?)
3) Proceed as in case 1. – (h,k)
   - sub in (x,y)
   - find a
   - write rule

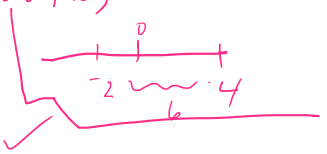
Example: Determine the rule of an absolute value function that passes through $(4,2)$, $(-2,2)$, and $(1,-10)$

$h = \frac{x_1 + x_2}{2}$

$h = \frac{4-2}{2}$

$h = \frac{2}{2}$

$h = 1$



Then I have confirmed the vertex is $(1,-10)$!

$f(x) = a|(x-h)| + k$

$2 = a|(4-1)| - 10$

$12 = a|3|$

$12 = 3a$

$4 = a$

Choose either of the other points to sub in for (x,y)

$f(x) = 4|(x-1)| - 10$

Case 3

If we are given at least two points on each ray, we can find

the rule of an absolute value function by:

1) Make a sketch! Plot the points.
2) Find the rule of both rays (lines)
3) System of equations to find the vertex (point of intersection)
4) Determine whether a is positive or negative
5) Determine the final rule.

Example: Determine the rule of an absolute value function passing through $(-1,4), (-3,-2), (4,2)$ and $(6,-4)$

Ray 1 $(4,2)$ and $(6,-4)$ are on the same line

$$a = \frac{y_2 - y_1}{x_2 - x_1}$$

$$a = \frac{-4-2}{6-4}$$

$$= \frac{-6}{2}$$

$$= -3$$

$y = ax + b$

$y = -3x + b$

$2 = -3(4) + b$

$2 = -12 + b$

$14 = b$

$y = -3x + 14$

Ray 2 the slope is the opposite of ray 1

$a = +3$

Sub in a point — either $(-3,-2)$ or $(-1,4)$

$4 = 3(-1) + b$

$7 = b$

$y = 3x + 7$

Vertex: $-3x + 14 = 3x + 7$

$7 = 6x$

$\frac{7}{6} = x$

Plug it in:

$y = 3\left(\frac{7}{6}\right) + 7$

$y = \frac{21}{2}$

$h, k = \frac{7}{6}, \frac{21}{2}$

$f(x) = -3\left|x - \frac{7}{6}\right| + \frac{21}{2}$

Case 4

If we are given two points on one ray and one on the other ray (that shares a y-value with one of the points on the other ray), we can:

1. Determine h
2. Determine a
3. Find the rule of one ray
4. Use h and the rule of one ray to solve for k

(5) Write the rule.

Example: Determine the rule of the function that passes through: $(-6,-7)$, $(-8,-11)$, and $(0,-11)$

Value of h between x values at $y = -11$ $\quad h = -4$

$$a = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-11-(-7)}{-8-(-6)}$$

$$= 2 \longrightarrow$$

* Check direction of the function: assign a sign.

In this case it is -2.

Determine the rule of the ray:

$y = ax + b$

$-11 = 2(-8) + b$

$5 = b$

$y = 2x + 5$ for one of the rays,

Find k: (insert value of h)

$y = 2(-4) + 5$

$y = -3 \quad \therefore k = -3$

$(h,k) = (-4,-3)$

$y = a|(x-h)| + k$

$y = -2|(x+4)| - 3$ ✓

Solving Absolute Value Functions

To solve for y – plug in x and solve (Reminder: the absolute value of a number is always positive).

To solve for x

1. Isolate the absolute value
2. Create two cases, one where $| \ | = +$ and one where $| \ | = -$
3. Solve for x in each case
4. * Check for extraneous answers (plug solution back into the equations)

Recall:

If $|x| = 2$

then $x = 2$ or $x = -2$

So, if $|x+6| = 2$

$x + 6 = 2$ $\quad x = -4$

$x + 6 = -2$ $\quad x = -8$

Ex. $2|x+4| - 2 = 0$ Check $x = -3$ Check $x = -5$

Ex. $2|x+4|-2=0$

$2|x+4|=2$

$|x+4|=1$

$x+4=1$ $\quad$ $x+4=-1$

$x=-3$ $\quad$ $x=-5$

Check $x=-3$

$2|-3+4|-2=0$

$2|1|-2=0$

$2-2=0$ ✓

TRUE

Check $x=-5$

$2|-5+4|-2=0$

$2|-1|-2=0$

$2(1)-2=0$

$2-2=0$ ✓

TRUE

∴ $2|x+4|-2=0$ when $x=-3$ and $x=-5$

Example: Solve $4|x+2|+6=1$

$4|x+2|=-5$

$|x+2|=\frac{-5}{4}$

No solutions.

Absolute value of a term can never be negative.

Ex. $f(x)=|x-2|-1$ and $g(x)=-2x$

Solve for where $f(x)=g(x)$

$|x-2|-1=-2x$

$|x-2|=-2x+1$

$+$ ↙ $\quad$ $-$ ↘

$x-2=-2x+1$ $\quad$ $x-2=-(-2x+1)$

$3x=3$ $\quad$ $x-2=2x-1$

$x=1$ $\quad$ $-x=1$

$x=-1$

Check $x=1$

$|1-2|-1=-2(1)$

Does $f(x)=g(x)$ where $x=1$

$1-1=-2$

$0=-2$

FALSE

NOT A SOLUTION

$x=-1$

$|-1-2|-1=-2(-1)$

$|-3|-1=2$

$3-1=2$

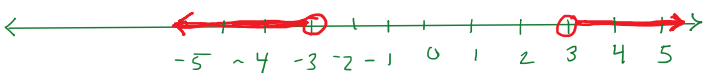
$2=2$

TRUE

∴ $f(x)=g(x)$ when $x=-1$

Solving absolute value inequalities

Recall $|x|>3$ would mean that x is more than 3 units away from 0.



so $|x|>3$, it means $x>3$ and $x<-3$ * NOTE change in direction of the inequality

To solve:

1. Isolate the absolute value
2. * Create two cases and remember to flip the inequality when you take the negative in the second case.
3. Solve. (Remember inequality rules! eg. flip the sign if you ÷ by a negative)

(4) Check using equality

(5) State solution.

Ex. Solve $|x-7|-4<-2$

$|x-7|<2$

$x-7<2$ and $x-7>-2$

$x-7<2$

$x<9$

$x-7>-2$

$x>5$

Check answers:

$x<9$, use $x=9$

$|9-7|-4=-2$

$|2|-4=-2$

$2-4=-2$

$-2=-2$

TRUE

$x>5$, $x=5$

$|5-7|-4=-2$

$|-2|-4=-2$

$2-4=-2$

$-2=-2$

TRUE

So $|x-7|-4<-2$ when x is <9 and >5.

$\therefore$ the solution is $]5,9[$

Solve: $|2x|-4\geq 0$

$|2x|\geq 4$

$2x\geq 4$ and $2x\leq -4$

$x\geq 2$ $\quad x\leq -2$

Check $x=2$

$|2\cdot 2|-4=0$ ✓

True

$x=-2$

$|-2\cdot 2|-4=0$ ✓

True

So, $|2x|-4\geq 0$ when x is greater than or equal to 2 and less than or equal to -2.

$\therefore$ the solution $]-\infty,-2]\cup[2,\infty[$

Given $f(x)=|x-2|$ and $g(x)=4x+8$, determine the interval over which $f(x)\geq g(x)$.

$|x-2|\geq 4x+8$

$x-2\geq 4x+8$

$-3x\geq 10$

$x\leq -\frac{10}{3}$

$x-2\leq -(4x+8)$

$x-2\leq -4x-8$

$5x\leq -6$

$x\leq -\frac{6}{5}$

Check $x\leq -\frac{10}{3}$

$\left|-\frac{10}{3}-2\right|=4\left(-\frac{10}{3}\right)+8$

$\left|-\frac{16}{3}\right|=-\frac{40}{3}+8$

$\frac{16}{3}=-\frac{40}{3}+\frac{24}{3}$

$\frac{16}{3}=-\frac{16}{3}$ NO

False; not a solution

Check $x\leq -\frac{6}{5}$

$\left|-\frac{6}{5}-2\right|=4\left(-\frac{6}{5}\right)+8$

$\left|-\frac{6}{5}-\frac{10}{5}\right|=-\frac{24}{5}+8$

$\left|-\frac{16}{5}\right|=-\frac{24}{5}+\frac{40}{5}$

$\frac{16}{5}=\frac{16}{5}$ ✓

TRUE

$f(x) \geq g(x)$ when $x \leq -\frac{6}{5}$ ∴ the solution is $]-\infty, -\frac{6}{5}]$

Equality

Given $f(x) = |x-3| + 2$ and $g(x) = -1|x-3| + 6$, solve $f(x) = g(x)$

Method 1, since $|x-3|$ is in both terms;

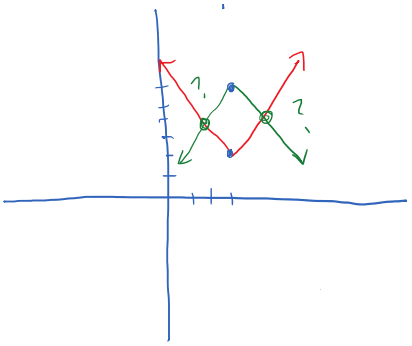
$1|x-3| + 2 = -1|x-3| + 6$

$2|x-3| = 4$

$|x-3| = 2$

$x - 3 = 2$ ↙ ↘ $x - 3 = -2$

$x = 5$ $\quad$ $x = 1$



Method 2

$f(x)$: left ray

$y = ax + b$

$y = -x + b$

$2 = -(3) + b$

$5 = b$

$y = -x + 5$

right ray

$y = ax + b$

$2 = 1(3) + b$

$y = x - 1$

$g(x)$: $y = 1x + b$

$y = 1x + b$

$6 = 1(3) + b$

$y = x + 3$

$y = -1x + b$

$6 = -1(3) + b$

$y = -x + 9$

$y = y$

$-x + 5 = x + 3$

$x = 1$

$x - 1 = -x + 9$

$x = 5$